Assignment III

$$x(x-1)\frac{d^2y}{dx^2} + (3x-1)\frac{dy}{dx} + y = 0$$

Soln.

a) $x(x-1)y'' + (3x-1)y' + y = 0$

Limit method...

$W_1^n + W_2^n + W_3^n = 0$

$W_1 = x(x-1)y''$

$v = x(x-1)$; $v' = 2x-1$, $v'' = 2$, $v''' = 0$

$u = y''$; $u' = y'''$, $u'' = y^{iv}$, $u''' = y^{v}$ ∴ $u^n = y^{n+2}$

$W_1^n = u^n v + n u^{n-1} v' + \frac{n(n-1)u^{n-2}}{2} v'' + \dots$

$= y^{n+2} \cdot [x(x-1)] + n y^{n+1} \cdot (2x-1) + \frac{n(n-1) \cdot y^n}{2} \cdot (2) + 0$

$= (x^2-x)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n$

$W_2 = (3x-1)y'$

$v = 3x-1$, $v' = 3$, $v'' = 0$

$u = y'$, $u' = y''$, $u'' = y'''$ ∴ $u^n = y^{n+1}$

$W_2^n = u^n v + n u^{n-1} v' + \dots$

$= y^{n+1} \cdot (3x-1) + ny^n \cdot 3 + 0$

$= (3x-1)y^{n+1} + 3ny^n$

$W_3 = y$

$v = 1$, $v' = 0$

$u = y$, $u' = y'$ ∴ $u^n = y^n$

$W_3^n = u^n v + 0$

$= y^n \cdot 1$

$= y^n$

∴ $W_1^n + W_2^n + W_3^n = 0$

$(x^2-x)y^{n+2} + (2x+1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$

Re-arranging to like terms:-

$\therefore (x^2-x)y^{n+2} + y^{n+1}[n(2x-1)+(3x-1)] + y^n[n(n-1)+3n+1] = 0$

~~$\therefore (x^2+x)y^{n+2} + y^{n+1}[n(2x-1)+(3x-1)] + y^n[n^2+2n+1]$~~

$(x^2-x)y^{n+2} + y^{n+1}[n(2x-1)+3x+1] + y^n[n^2+2n+1] = 0$

~~$\therefore (x^2-x)y^{n+2} = -[y^n \cdot [y(n(2x-1)+3x+1)] + y$~~

$y^{n+2}(x^2-x) = -y^{n+1}[n(2x-1)+3x-1] - y^n[(n+1)(n+1)]$

$y^{n+2}(x^2-x) = -y^{n+1}[n(2x-1)+3x-1) - y^n(n+1)^2.$

at $x = 0$

From original eqn.

$(0)y'' + (3(0)-1)y' + y = 0$

$-y'_0 = -y$

$y'_0 = y$ . . . eqn ①

x^1 * when $x=0$, $n=0$

$y''(0) = -y'(-1) - y^0(1)^2$

$y'_0 = 1$

x^2 * when $x=0$, $n=1$

$y'''(0) = -y''(1(-1)-1) - y'(2)^2$

$0 = 2y'' - 4y'$

$y''_0 = 2y'$, $y' = 1$ $\therefore y'' = 2$

x^3 * when $x=0$, $n=2$

$y^{iv}_0(0) = -y'''_0(2(-1)-1) - 9y''_0$

$0 = 3y'''_0 - 9y''_0$

$9y''_0 = 3y'''_0$

$y'''_0 = 3y''_0$

$y'''_0 = 3[2y']$

$y'''_0 = 3!\, y'_0$

x^4 * when $x=0$, $n=3$

$y^{v}_{(0)} = y^{iv}_{(0)}(-3-1) - 16y'''_0$

$=0$

$0 = 4y_0^{iv} - 16y_0^{'''}$

$\therefore 16y_0^{'''} = 4y_0^{iv}$

$\therefore y_0^{iv} = 4y_0^{'''}$

$y_0^{iv} = 4[3!y_0']$

$y_0^{iv} = 4!y_0'$

$x^5 \rightarrow$ when $x = 0, n = 4$

$y_0^{vi}(0) = -y_0^{v} \cdot (-4-1) - 25y^{iv}$

$0 = 5y_0^{v} - 25y_0^{iv}$

$25y_0^{iv} = 5y_0^{v}$

$y_0^{v} = 5y_0^{iv}$

$\therefore y_0^{v} = 5(4!y_0')$

$y_0^{v} = 5!y_0'$

$x^6 \rightarrow$ when $x = 0, n = 5$

$y_0^{vii}(0) = -y_0^{vi}(-5-1) - y_0^{v}(36)$

$0 = 6y_0^{vi} - 36y_0^{v}$

$36y_0^{v} = 6y_0^{vi}$

$y_0^{vi} = 6y_0^{v}$

$\therefore y_0^{vi} = 6(5!y_0')$

$y_0^{vi} = 6!y_0'$

$x^7 \rightarrow$ when $x = 0, n = 6$

$y_0^{viii}(0) = y_0^{vii}(-6-1) - y_0^{vi}(49)$

$0 = -y_0^{vii}(-7) - 49y_0^{vi}$

$\therefore 49y_0^{vi} = 7y_0^{vii}$

$y_0^{vii} = 7y_0^{vi}$

$y_0^{vii} = 7(6!y_0')$

$y_0^{vii} = 7!y_0'$

Applying Leibnitz maclaurin's formula

$y = y_0 + x(y_0') + \frac{x^2}{2!}(y_0'') + \frac{x^3}{3!}(y_0''') + \frac{x^4}{4!}(y_0^{iv}) + \frac{x^5}{5!}(y_0^{v}) + \frac{x^6}{6!}(y_0^{vi}) + \frac{x^7}{7!}(y_0^{vii}) + \cdots$

$= y_0 + x(y_0') + \frac{x^2}{2!}(2y_0') + \frac{x^3}{3!}(3!y_0') + \frac{x^4}{4!}(4!y_0') + \frac{x^5}{5!}(5!y_0') + \frac{x^6}{6!}[6!y_0'] + \frac{x^7}{7!}[7!y_0'] + \cdots$

$= y_0 + xy_0' + x^2y_0' + x^3y_0' + x^4y_0' + x^5y_0' + x^6y_0' + x^7y_0' + \cdots$

$= y_0 + y_0'(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 \cdots)$

but $y_0' = y_0$ (eqn 1)

$= y_0 + y_0(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$= y_0(1 + (x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7))$

b) $x = 5$ and $y_0'(0) = 0.0005$

recall $y_0' = y_0$

$\therefore y_5 = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7]$

$y_5 = 0.0005(97,656)$

$y_5 = 48.828$

$x = 8$

$y_8 = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$y_8 = 0.0005(2,396,745)$

$= 1198.3725.$

$x = 10m$

$y_{10} = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$= 0.0005(11,111,111)$

$= 5555.5555.$

c) Command window

```
clear
clc
y0 = 0.0005
x = (0:10)
y = y0 .* (1+x+x^2+x^3+x^4+x^5+x^6+x^7)
yn = sub(x, y)
Plot (x, yn)
grid on
grid minor
xlabel ('x')
y label ('structural element')
```

Graph of Structural Element Against x.