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ENG 381 ENG MATHS (~~xxx~~ ASSIGNMENT)

The model for deformation (y) of a structural element is represented by the expression given as $x(x-1)y'' + (3x-1)y' + y = 0$.

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz - Maclaurin ~~method~~:

a) obtain the power series solution of the model up to and including the term in x^7

b) estimate the approximate deformation when $x = 5, 8$ and $10m$ and

c) with the aid of a MATLAB mfile program plot the response of the structural element for $0 \le x \le 10m$.

Solution

a) $x(x-1)y'' + (3x-1)y' + y = 0$

$\Rightarrow (x^2-x)y'' + (3x-1)y' + y = 0$ Expanding the bracket

we can then assume

$(x^2-x)y'' = w_1$ and $y = w_3$

$(3x-1)y' = w_2$

$\therefore$ Using Leibnitz theorem $= u^n v + n u^{n-1} v' + \frac{n(n-1)u^{n-2}v''}{2!} + \dots$

$\therefore w_1$

Let $u = y''$ $v = x^2 - x$
$u^n = y^{n+2}$ $v' = 2x - 1$
$u^{n-1} = y^{n+1}$ $v'' = 2$
$u^{n-2} = y^n$ $v''' = 0$

w_2

let $u = y'$ $v = 3x - 1$
$u^n = y^{n+1}$ $v' = 3$
$u^{n-1} = y^n$ $v'' = 0$

w_3

let $u = y$ $v = 1$
$u^n = y^n$ $v' = 0$

$\therefore$ Summing w_1, w_2 and w_3 we would have $(w_1 + w_2 + w_3)$ but $w_1 = (y^{n+2} \cdot (x^2-x)) + n(y^{n+1} \cancel{2x-1}) + \frac{n(n-1)y^n \cdot \cancel{2}}{\cancel{2!}}$

$w_2 = [y^{n+1} \cdot (3x-1)] + n y^n \cdot 3 + 0$ and

$w_3 = y^n$

$\therefore w_1 + w_2 + w_3 = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + n(n-1)y^n + y^{n+1}(3x-1) + n y^n 3 + y^n = 0$

$= y^{n+2}(x^2-x) + y^{n+1}(2x-n) + y^{n+1}(3x-1) + n(n-1)y^n + n y^n 3 + y^n = 0$

assuming $x = 0$

$= \cancel{y^{n+2}(0-0)}^{0} + y^{n+1}(2(0)-n) + y^{n+1}(3(0)-1) + n(n-1)y^n + n y^n \cdot 3 + y^n = 0$

$= - n y^{n+1} - y^{n+1} + y^n(n^2-n) + 3n y^n + y^n = 0$

Collecting like terms

$$y^{(n+1)}(-n-1) + y^{n}(n^2-n+3n-1) = 0$$

$$-y^{(n+1)}(n+1) + y^{n}(n^2+2n-1) = 0$$

$$y^{(n+1)}(n+1) = y^{n}(n^2+2n-1)$$

$$y^{(n+1)}(n+1) = y^{n}(n+1)(n+1) \quad \text{Expanding the bracket}$$

d bs by $(n+1)$

$$y^{(n+1)} = y^{n}(n+1) \quad \text{is thus the reaccurence relation}$$

$$(y^{n+1})_0 = y^{n}(n+1)_0$$

$$(y^{0})_0 = 0.0005$$

$$(y^{1})_0 = 0.0005$$

thus when $n=0$ $\cancel{y^{0+1}}$

$$(y^{(0+1)})_0 = (y^{(0)})_0 (0+1)$$

$$(y^{1})_0 = (y^{0})_0 \, 1$$

when $n=1$

$$(y^{(1+1)})_0 = (y^{1})_0 (1+1)$$

$$(y^{2})_0 = (y^{1})_0 \, 2 = 2(y^{0})_0$$

when $n=2$

$$(y^{2+1})_0 = (y^{2})_0 (2+1)$$

$$(y^{3})_0 = 3(y^{2})_0 = 6(y^{1})_0$$

when $n=3$

$$(y^{(3+1)})_0 = (y^{3})_0 (3+1)$$

$$(y^{4})_0 = (y^{3})_0 \, 4 = 24(y^{1})_0$$

when $n=4$

$$(y^{5})_0 = (y^{4})_0 \, 5 = 120(y^{1})_0$$

when $n=5$

$$(y^{6})_0 = (y^{5})_0 \, 6 = 720(y^{1})_0$$

When $n=6$

$$(y^{7})_0 = (y^{6})_0 \, 7 = 5040(y^{1})_0$$

Using Maclaurus Series

$$y = (y^{0})_0 + x(y^{1})_0 + \frac{x^2}{2!}(y^{2})_0 + \frac{x^3}{3!}(y^{3})_0 + \frac{x^4}{4!}(y^{4})_0 + \dots$$

$$y = (y^{0})_0 + x(y^{0})_0 + \frac{x^2}{\cancel{2!}}\cancel{2}(y^{1})_0 + \frac{x^3}{\cancel{3!}}\cancel{6}(y^{1})_0 + \frac{x^4}{\cancel{4!}}(\cancel{24}\,y^{1})_0 + \frac{x^5}{\cancel{5!}}\cancel{120}(y^{1})_0 +$$

$$\frac{x^6}{\cancel{6!}}\cancel{720}(y^{1})_0 + \frac{x^7}{\cancel{7!}}\cancel{5040}(y^{1})_0 + \dots$$

$\therefore$

$y = (y^\circ)_0 (x+1) + (x^2+x^3+x^4+x^5+x^6+x^7)(y')$

$\therefore y = 0.0005(x+1) + (x^2+x^3+x^4+x^5+x^6+x^7)\,0.0005$

ii) estimating the approximate deformation when $x = 5, 8$ and $10m$

when $x = 5m$

$y = 0.0005(5+1) + (5^2+5^3+5^4+5^5+5^6+x^7)\,0.0005$

$\therefore y = 48.828m$.

when $x = 8$

$y = 0.0005(8+1) + (8^2+8^3+8^4+8^5+8^6+8^7)\,0.0005$

$\therefore y = 1198.373m$

when $x = 10$

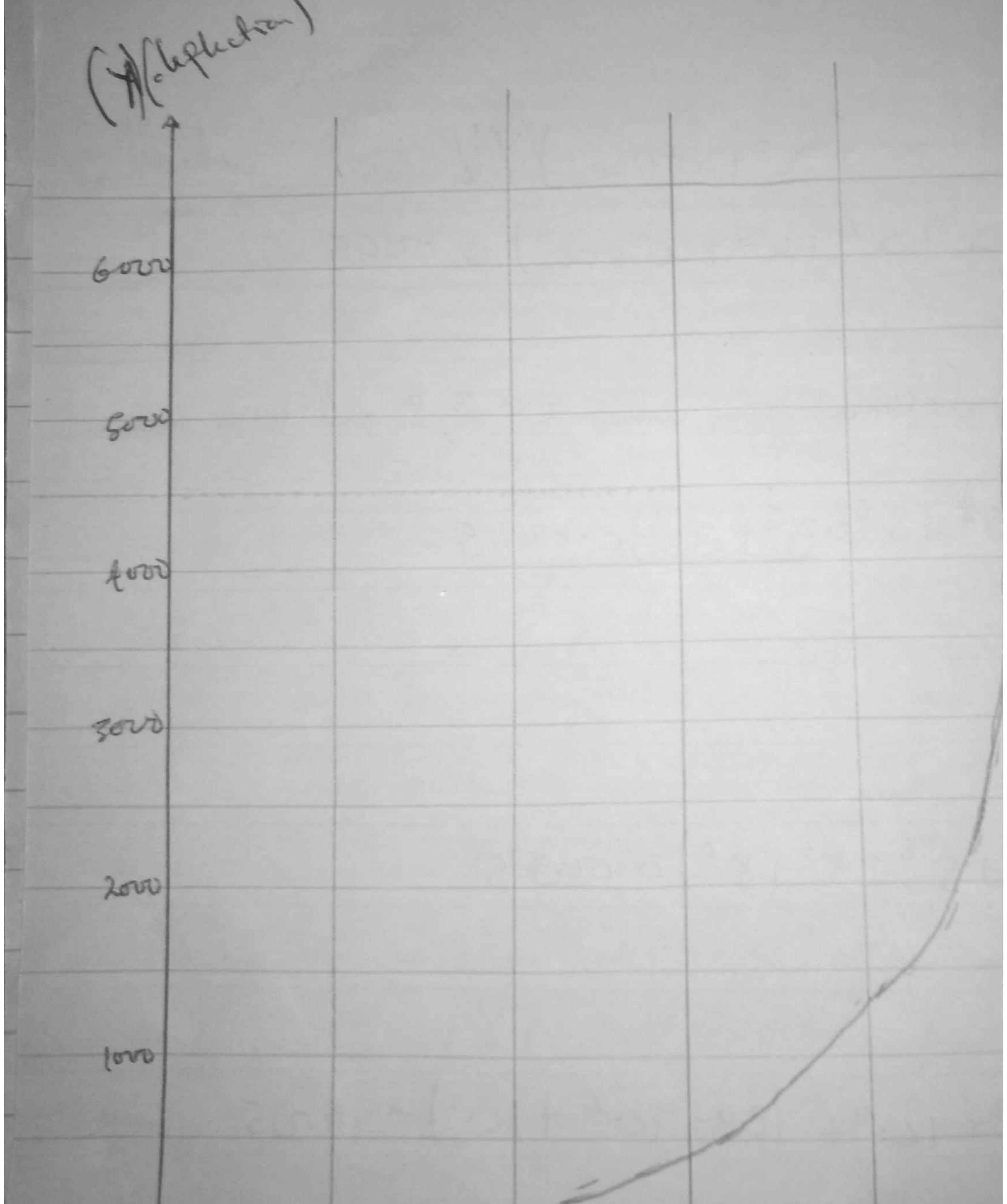
$y = 0.0005(10+1) + (10^2+10^3+10^4+10^5+10^6+10^7)\,0.0005$

$\therefore y = 5555.56m$.

iii) MATLAB mfile

Comand window

```
Clear
clc
Close all
x = 0:0.01:10
Y = (0.0005*(x+1)) +((x^2+x^3+x^4+x^5+x^6+x^7)*
0.0005)
Yn= subs (Y)
plot (x, Yn)
x label ('m')
Y label ('Deflection')
axis tight
grid on
grid minor
```


(deflection)
6000
5000
4000
3000
2000
1000