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DEPARTMENT: COMPUTER ENGINEERING

ASSIGNMENT ENG 281

1. $y = 3e^{2x}$ and $y = 3e^{-x}$ at the points $x = 1$ and $x = 2$
Find the area bounded by the curves

solution

Area between 2 curves

$A = \int_a^b f(x) - g(x)\,dx$ where $f(x) = 3e^{2x}$ and $g(x) = 3e^{-x}$

$a = 1$, $b = 2$

$$A = \int_1^2 3e^{2x} - (3e^{-x})\,dx$$

$$A = \int_1^2 3e^{2x} - 3e^{-x} = 3\int_1^2 e^{2x} - e^{-x}$$

$$A = 3\left[\frac{e^{2x}}{2} + e^{-x}\right]_1^2$$

$$A = 3\left[\left(\frac{e^{2(2)}}{2} + e^{-2}\right) - \left(\frac{e^{2(1)}}{2} + e^{-1}\right)\right]$$

$$A = 3\left[(27.2 + 0.135) - 4.08\right]$$

$$A = 3(23.275)$$

$$A = 69.823 \text{ units}^2 \approx 70 \text{ units}^2$$

2. $y = 2\sin\frac{\pi}{10}t$ and $x = 2 + 2t - 2\cos\frac{\pi}{10}t$ $t = 0$ and $t = 12$

Find the area bounded by the curve

$$A = \int_a^b y\,dx$$

$y = 2\sin \frac{\pi}{10} t$ $\qquad x = 2 + 2t - 2\cos \frac{\pi}{10} t$

$$A = \int_a^b y \, dx$$

let $\frac{\pi}{10} = u$

$x = 2 + 2t - 2\cos \frac{\pi}{10} t$

$\frac{dx}{dt} = 2 + 2u \sin ut$

$dx = 2 + 2u \sin ut \, dt$

$$A = \int_0^{10} 2\sin ut \, (2 + 2u \sin ut) \, dt$$

$$= 4 \int_0^{10} \sin ut + u \sin^2 ut \, dt$$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$$= 4 \int_0^{10} \sin ut + \frac{u}{2}(1 - \cos 2ut) \, dt$$

$$= 4 \left[-\frac{\cos ut}{u} + \frac{ut}{2} - \frac{u}{4u} \sin 2ut \right]_0^{10}$$

$c = \frac{\pi}{10}$

$$= 4 \left[\frac{\pi t}{20} - \frac{10}{\pi} \cos \frac{\pi}{10} t - \frac{1}{4} \sin \frac{\pi}{5} t \right]_0^{10}$$

$$= \left[\frac{\pi t}{5} - \frac{40}{\pi} \cos \frac{\pi t}{10} - \sin \frac{\pi t}{5} \right]_0^{a}$$

$$= \left(\frac{\pi(10)}{5} - \sin \frac{\pi(10)}{5} - \frac{40}{\pi} \cos \frac{\pi(10)}{10} \right) - \left(\frac{\pi(0)}{5} - \sin \frac{\pi(0)}{5} - \frac{40}{\pi} \cos \frac{\pi(0)}{10} \right)$$

$= 2\pi + \frac{40}{\pi} + \frac{40}{\pi} = 31.75$ square units