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ENG 381 Assignment III

1) The model for the deformation (y) of a structural filament is represented by the ~~equation~~ expression given in equation (1)

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005m$ applying leibnitz-maclaurin's theorem

a) Obtain the power series of the model up to and including the term in x^7.

b) estimate the approximate deformation when $x = 5, 8$ & 10 m and

c) with the aid of a MATLAB ~~mfile~~ m file program, plot the response of the structural filament for $0 \leq x \leq 10m$.

Solu.

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Let $w_1 = x(x-1)y''$

$w_2 = (3x-1)y'$

$w_3 = y$.

$w_1 = x(x-1)y''$

$u = y''$ $\quad v = x^2 - x$

$u^n = y^{(n+2)}$ $\quad v^{(1)} = 2x - 1$

$u^{(n-1)} = y^{(n+1)}$ $\quad v^{(2)} = 2$

$y^{(n-2)} = y^n$ $\quad v^{(3)} = 0$.

$w_2 = (3x-1)y'$

$u = y'$ $\quad v = 3x - 1$

$u^n = y^{(n+1)}$ $\quad v^{(1)} = 3$

$u^{(n-1)} = y^{(n)}$ $\quad v^{(2)} = 0$

$w_3 = y$

$u = y$ $\quad v = 1$

$u^n = y^n \quad v^n = 0$

For w_1

$w_1 = y^{(n+2)} \cdot (x^2 + x) + n \cdot y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2!} \cdot y^n \cdot 2 + \dots$

$w_1 = (x^2+x)y^{(n+2)} + n(2x-1)y^{(n+1)} + n(n-1)y^n$

For w_2

$w_2 = y^{(n+1)} \cdot 3x - 1 + n \cdot y^{(n)} \cdot 3 + \dots$

$= (3x-1)y^{(n+1)} + 3ny^{(n)} \dots$

For w_3

$= y^n \cdot 1$

$= y^n$

Adding $w_1 + w_2 + w_3$

$(x^2+x)y^{(n+2)} + n(2x-1)y^{(n+1)} + n(n-1)y^n + (3x-1)y^{(n+1)} + y^n + 3ny^{(n)} = 0$

$y^n + 3ny^{(n)} = 0$

Substitute $x = 0$

$y^n + 0 \cdot y^{(n+2)} + n(-1)y^{(n+1)} + n(n-1)y^n + 0 \cdot y^{(n+1)} + 3ny^{(n)} = 0$

$y^n + n(n-1)y^{(n)} + 3ny^{(n)} = 0$

$y^n(1 + n(n-1) + 3n) = 0$

$-y^{(n+1)} + y^n - ny^{(n+1)} + n(n-1)y^n + 3ny^{(n)} = 0$

$y^{(n+1)}(-1-n) + y^n(1 + n(n-1) + 3n) = 0$

$y^{(n+1)} = \frac{-y^n(1 + n^2 - n + 3n)}{-(1+n)}$

$y^{(n+1)} = \frac{y^n(1 + n^2 + 2n)}{(1+n)}$

$(y^{(n+1)})_0 = \frac{y^n(n+1)(n+1)}{(n+1)}$

$(y^{(n+1)})_0 = y^n(n+1)$

$n=0, (y^{(1)})_0 = (y^0)_0$

when $n=1$, $(y^{(2)})_0 = (y^1)_0 2 = 2(y^0)_0$ or $2(y^{(0)})_0$

when $n=2$, $(y^{(3)})_0 = 3(y^{(2)})_0 = 3 \times 2(y^{(1)})_0 = 3(y^{(1)})_0$

when $n=3$, $(y^{(4)})_0 = 4(y^{(3)})_0 = 4 \times 3 \times 2(y^{(1)})_0 = 4!(y^{(1)})_0$

when $n=4$, $(y^{(5)})_0 = 5(y^{(4)})_0 = 5 \times 4!(y^{(1)})_0 = 5!(y^{(1)})_0$

when $n=5$, $(y^{(6)})_0 = 6(y^{(5)})_0 = 6 \times 5!(y^{(1)})_0 = 6!(y^{(1)})_0$

" $n=6$, $(y^{(7)})_0 = 7(y^{(6)})_0 = 7 \times 6!(y^{(1)})_0 = 7!(y^{(1)})_0$

when $n=7$, $(y^{(8)})_0 = 8(y^{(7)})_0 = 8\times 7!\,(y^{(6)})_0 = 8!\,(y^{(6)})_0$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}\cdot 2(y^{(2)})_0 + \frac{x^3}{3!}\cdot 3!\,(y^{(3)})_0 + \frac{x^4}{4!}\cdot$

$+\frac{x^5}{5!}\cdot 5!\,(y^{(5)})_0 + \frac{x^6}{6!}\cdot 6!\,(y^{(6)})_0 + \frac{x^7}{7!}\cdot 7!\,(y^{(7)})_0 + \frac{x^8}{8!}\cdot$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + x^2(y^{(2)})_0 + x^3\cdot(y^{(3)})_0 + x^4(y^{(4)})_0 + x^5(y^{(5)})_0$

$+x^6(y^{(6)})_0 + x^7(y^{(7)})_0 + x^8(y^{(8)})_0$

$y = (y^{(0)})_0(1+x+x^2+x^3+x^4+x^5+x^6+x^7)$

b) when $x = 5$, 8, and 10m.

$(y^{(0)})_0 = 0.0005$m

$(y'^{(0)})_0 = 0.0005$

$y = (y^{(0)})_0(1+x) + (y^{(1)})_0(x^2+x^3+x^4+x^5+x^6+x^7)x$

b) when $x = 5, 8, 10$m

$(y^{(0)})_0 = 0.0005$m

$(y'^{(0)})_0 = 0.0005$

when $x = 5$

$y = 0.0005\text{m}(1+5) + (0.005)(5^2+5^3+5^4+5^5+5^6+5^7)$

$y = 0.0005\text{m} + 0.0025\text{m} + 48.825\text{m}$

$y = 48.825\text{m} + 0.0005\text{m} + 0.0025\text{m}$

$y = 48.828\text{m}$

when $x = 8$m

$y = 0.0005\text{m}(1+8) + 0.0005(8^2+8^3+8^4+8^5+8^6+8^7)$

$y = (1198.37 + 0.0005 + 4\times10^{-3})\text{m}$

$y = 1198.37\text{m}$

when $x = 10$m

$y = 0.0005\text{m}(1+10) + 0.0005(10^2+10^3+10^4+10^5+10^6+10^7)$

$y = 5.5\times10^{-3} + 0.0005(11111100)$

$y = 5.5\times10^{-3} + 5555.55$

$y = 5555.556$m.

Commandwindow

```
- Clear all
- clc.
- Close all.
- syms x.
x = 0:0.01:10
y = 0.0005*(1+x) + ((x^2)+(x^3)+(x^4)+(x^5)+(x^6)+(x^7))*(0.0005)).
plot(x,y)
- grid minor
- grid ~~major~~ on
- xlabel('time')
- ylabel('Decay')
```