ATOGWE VICTORIA ALOIYE.

18/ENG08/003.

BIOMEDICAL ENGINEERING.

ENG 281.

1. Find the area bounded by the curves $y = 3e^{2x}$ and $y = 3e^{-x}$ and the ordinates at $x = 1$ & $x = 2$.

2. The parametric equations of a curve are $y = 2\sin \pi/10t$ & $x = 2 + 2t - \cos \pi/10t$. Find the mean under the curve between $t = 0$ & $t = 10$.

Solution.

1. $y_1 = 3e^{2x}$ $\qquad y_2 = 3e^{-x}$

$$A = \int_1^2 y_1 - y_2 \, dx$$
$$= \int_1^2 3e^{2x} - 3e^{-x} \, dx$$
$$= 3\int_1^2 e^{2x} - e^{-x} \, dx$$
$$= 3\int_1^2 \frac{e^{2x}}{2} - \cancel{1}(-e^{-x}) dx$$
$$= 3\int_1^2 \frac{e^{2x}}{2} + e^{-x} \, dx$$
$$= 3 \Big|_1^2 \frac{e^{2x}}{2} + e^{-x}$$
$$= 3\left[\frac{e^{2(2)}}{2} + e^{-2}\right] - 3\left[\frac{e^{2(1)}}{2} + e^{-1}\right]$$
$$= (3 \times \cancel{22} \, 27.4344103) - (3 \times 4.062407411)$$
$$= 82.30 - 12.18$$
$$= 70.12 \text{ square unit.}$$

2.

2. $A = \int_a^b y \, dx$

But $\frac{dx}{dt} = 2 + 2(\pi/10) \sin \pi/10 t$

$dx = 2 + 2(\pi/10)\sin(\pi/10t) \cdot dt$

$\therefore A = \int_0^{10} 2\sin(\pi/10t)(2 + 2(\pi/10)\sin(\pi/10t)) \cdot dt$

$A = \int_0^{10} (4\sin(\pi/10t) + 4(\pi/10)\sin^2(\pi/10t)) \cdot dt$

$A = 4\int_0^{10} (\sin(\pi/10t) + (\pi/10)\sin^2(\pi/10t)) \cdot dt$

By trigonometric identities:

$\sin^2(\pi/10t) = 1/2\,(1 - \cos 2(\pi/10t))$ --

$\therefore A = 4\int_0^{10} (\sin(\pi/10t) + (\pi/10)(1/2(-\cos 2(\pi/10t) + 1))) \cdot dt$

$A = 4\int_0^{10} \left(\sin(\pi/10t) + (\pi/10)\left(\frac{-\cos 2(\pi/10t)}{2} + \frac{1}{2}\right)\right) \cdot dt$

$A = 4\left[\frac{-10}{\pi}\cos\left(\frac{10}{\pi}t\right) + \frac{\pi}{20}t + -\left(\frac{\pi}{20} \cdot \frac{5}{\pi}\sin 2\left(\frac{\pi}{10}t\right)\right)\right]_0^{10}$

$A = \left[\frac{-40}{\pi}\cos\left(\frac{\pi}{10}t\right) + \frac{\pi}{5}t - \sin 2\left(\frac{\pi}{10}t\right)\right]_0^{10}$

~~At~~ Taking $\pi = 22/7$:

$\therefore A = \left[\frac{-40}{22/7}\cos\left(\frac{22/7}{10}t\right) + \frac{22/7}{5}t - \sin 2\left(\frac{22/7}{10}t\right)\right]_0^{10}$

$A = \left[\frac{-40}{22/7}\cos\left(\frac{22/7}{10}(10)\right) + \frac{22/7}{5}(10) - \sin 2\left(\frac{22/7}{10}(10)\right)\right]_0^{10} -$

$\left[\frac{-40}{22/7}\cos\left(\frac{22/7}{10}(0)\right) + \frac{22/7}{5}(0) - \sin 2\left(\frac{22/7}{10}(0)\right)\right]$

$A = [-12.7081 + 6.2857 - 0.1095] - [-12.7273 + 0 - 0]$

$A = -12.7081 + 6.2857 - 0.1095 + 12.7273$

$A = \underline{6.1954 \text{ square units}}$