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1) $y = 3e^{2x}$ and $y = 3e^{-x}$ at the points $x=1$ and $x=2$. find the area bounded by the curve

Solution

Area between two curves

$A = \int_a^b f(x) - g(x)\,dx$ where $f(x) = 3e^{2x}$ and $g(x) = 3e^{-x}$, $a=1$, $b=2$

$A = \int_1^2 3e^{2x} - (3e^{-x})\,dx$

$A = \int_1^2 3e^{2x} - 3e^{-x} = 3\int_1^2 e^{2x} - e^{-x}$

$A = 3\left[\frac{e^{2x}}{2} + e^{-2}\right]_1^2$

$A = 3\left[\left(\frac{e^{2(2)}}{2} + e^{-2}\right) - \frac{e^{2(1)}}{2} + e^{-1}\right]$

$A = 3\left[(27.2 + 0.135) - 4.06\right]$

$A = 3(23.275$

$= 69.823$ units $\simeq 70$ units

2) $y = 2\sin\frac{\pi}{10}t$ and $x = 2+2t-2\cos\frac{\pi}{10}t$. $t=0$ and $t=10$. find the area bounded by the curve

Solution

$A = \int_a^b y\,dx$

$A = \int_0^{10} 2\sin\frac{\pi}{10}t\,dx$

$x = 2+2t-2\cos\frac{\pi}{10}t$

$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi}{10}t$

$dx = 2 + \frac{\pi}{5}\sin\frac{\pi}{10}t\,dt$

2) $y = 2\sin \pi/10\, t$ $\quad x = 2 + 2t - 2\cos \pi/10\, t$

$$A = \int_a^b y\, dx$$

let $\pi/10 = u$

$x = 2 + 2t - 2\cos \pi/10\, t$

$\frac{dx}{dt} = 2 + 2u\sin ut$

$dx = 2 + 2u\sin ut\, dt$

$$A = \int_0^{10} 2\sin ut\,(2 + 2u\sin ut)\, dt$$

$$= 4\int_0^{10} \sin ut + u\sin^2 ut\, dt$$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$$= 4\int_0^{10} \sin ut + \frac{u}{2}(1 - \cos 2ut)\, dt$$

$$= 4\left[\frac{-\cos ut}{u} + \frac{ut}{2} - \frac{u}{4u}\sin 2ut \right]_0^{10}$$

$C = \pi/10$

$$= 4\left[\frac{\pi t}{20} - \frac{10}{\pi}\cos \frac{\pi}{10} t - \frac{1}{4}\sin \frac{\pi}{5} t \right]_0^{10}$$

$$= \left[\frac{\pi t}{6} - \frac{40}{\pi}\cos \frac{\pi t}{10} - \frac{\sin \pi t}{5} \right]_0^{10}$$

$$= \left(\frac{\pi(10)}{5} - \frac{\sin \pi(10)}{5} - \frac{40}{\pi}\cos \frac{\pi(10)}{10} \right) - \left(\frac{\pi(0)}{6} - \frac{\sin \pi(0)}{5} - \frac{40}{\pi}\cos \frac{\pi(0)}{10} \right)$$

$= 2\pi + 40/\pi + 40/\pi = 31.75$ square units