

1 (a) $x(x-1)y'' + (3x-1)y' + y = 0$

For the nth derivative

$$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$x(x-1)y^{n+2} + y^{n+1}(2xn - n + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$$

At $x=0$,

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 + 2n + 1) = 0$$

$$\therefore -(n+1)(y^{n+1})_0 = -(n^2 + 2n + 1)(y^n)_0$$

$$(n+1)(y^{n+1})_0 = (n^2 + 2n + 1)(y^n)_0$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

At $n=0$, $(y^{0+1})_0 = (0+1)(y^0)_0$

$$(y^1)_0 = (y^0)_0$$

At $n=1$, $(y^{1+1})_0 = (1+1)(y^1)_0$

$$(y^2)_0 = 2(y^1)_0$$

At $n=2$, $(y^{2+1})_0 = (2+1)(y^2)_0$

$$\therefore (y^3)_0 = 3(y^2)_0$$

$$(y^3)_0 = 6(y^1)_0$$

At $n=3$, $(y^{3+1})_0 = (3+1)(y^3)_0$

$$(y^4)_0 = 4(y^3)_0 = 4 \times 6(y^1)_0 = 24(y^1)_0$$

At $n=4$, $(y^5)_0 = (4+1)(y^4)_0$

$$= 5(y^4)_0$$

$$= 5 \times 24(y^1)_0 \therefore (y^5)_0 = 120(y^1)_0$$

At $n=5$, $(y^6)_0 = (5+1)(y^5)_0$

$$= 6(y^5)_0$$

$$= 6 \times 120(y^1)_0 \therefore (y^6)_0 = 720(y^1)_0$$

At $n=6$, $(y^7)_0 = (6+1)(y^6)_0$

$$= 7(y^6)_0$$

$$= 7 \times 720(y^1)_0 \therefore (y^7)_0 = 5040(y^1)_0$$

From Maclaurin series.

$$y = (y)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$$

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(2(y')_0) + \frac{x^3}{3!}(6(y')_0) + \frac{x^4}{4!}(24(y')_0) + \frac{x^5}{5!}(120(y')_0) + \frac{x^6}{6!}(720(y')_0) + \frac{x^7}{7!}(5040(y')_0) + \dots$$

$$\therefore y = (y)_0 + x(y')_0 + x^2(y')_0 + x^3(y')_0 + x^4(y')_0 + x^5(y')_0 + x^6(y')_0 + x^7(y')_0 + \dots$$

$$y = (y)_0 + (y')_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

Recall $(y)_0 = 0.0005\text{m}$ and $(y')_0 = 0.0005$

$$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

b) when $x = 5\text{m}$

$$y = 0.0005 + 0.0005 [5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$

$$y = 0.0005 + 0.0005 [97655]$$

$$y = 48.828\text{m}$$

when $x = 8\text{m}$

$$y = 0.0005 + 0.0005 [8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152]$$

$$y = 0.0005 + 0.0005 [2396744]$$

$$y = 1198.3725\text{m}$$

when $x = 10\text{m}$

$$y = 0.0005 + 0.0005 [10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000]$$

$$y = 0.0005 + 0.0005 [11111110]$$

$$y = 5555.5555\text{m}$$

command window

clear

clc

Syms x, y

x = [0:10];

y = 0.0005 + 0.0005 [x + (x^2) + (x^3) + (x^4) + (x^5) + (x^6) + (x^7)];

plot(x, y);

grid on;

grid minor;

xlabel('x')

ylabel('structural deformation')