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Maths Assignment
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Solution

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t \quad \text{---- (i)}$$

---- in aux form

$Y = C.F + P.I$

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+2)(m+3) = 0$

$m_1 = -2$ & $m_2 = -3$.

$\therefore x = Ae^{-2t} + Be^{-3t}$

Solving for P.I

$x = P.I$.

$x = C\cos t + A\sin t$.

$\frac{dx}{dt} = -C\sin t + A\cos t$.

$\frac{d^2x}{dt^2} = -C\cos t - A\sin t$

Substituting back in eqn (i)

$-C\cos t - A\sin t + 5[-C\sin t + A\cos t] + 6[C\cos t + A\sin t] = \cos t$

$-C\cos t - A\sin t - 5C\sin t + 5A\cos t + 6C\cos t + 6A\sin t = \cos t$

$-C\cos t + 6C\cos t - A\sin t + 6A\sin t - 5C\sin t + 5A\cos t = \cos t$

$5C\cos t + 5A\sin t - 5C\sin t + 5A\cos t = \cos t$.

$5C\cos t + 5A\cos t + 5A\sin t - 5C\sin t = \cos t$.

$5C\cos t + 5A\cos t = \cos t$

$5C + 5A = 1$ ----- (i)

$5A\sin t - 5C\sin t = 0$

$5A - 5C = 0$ --- (ii)

applying simultaneous eqn.

$$5C + 5A = 1$$
$$+ \quad -5C + 5A = 0$$
$$10A = 1$$
$$\therefore A = \frac{1}{10}$$

Sub A in eqn (i)

$$5C + 5(1/10) = 1$$
$$5C + 1/2 = 1$$
$$5C = 1 - 1/2.$$
$$5C = 1/2.$$
$$C = 1/2 \times \frac{1}{5}$$
$$C = \frac{1}{10}$$

$$\therefore y = \frac{1}{10}[\sin t + \cos t]$$

$$G.S = Ae^{-2t} + Be^{-3t} + \frac{1}{10}[\sin t + \cos t]$$

when $t = 0$, $x = 0.1$

$$0.1 = A + B + 0 + 1/10$$
$$A + B = 0 \quad \text{---- (i)}$$

when $t = 0$, $\frac{dx}{dt} = 0$.

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t).$$
$$0 = -2A - 3B + 0.1$$
$$-0.1 = -2A - 3B. \quad \text{---- (ii)}$$

remember $A + B = 0$ ---- (iii)

$$A = -B \quad \text{---- (iv)}$$
$$-0.1 = -2(-B) - 3B.$$
$$-0.1 = 2B - 3B.$$
$$-0.1 = -B.$$
$$B = 0.1$$

Knowing that $A = -B$.

$A = -0.1$

$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + 1/10[\sin t + \cos t]$.

$x = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}[\sin t + \cos t]$

2. ⇒ Command window
⇒ Clear
⇒ Clc
⇒ Close all
⇒ Sym t.
⇒ x = [0.1*exp(-3*t) - 0.1*exp(-2*t) + 0.1*cos(t) + 0.1*sin(t)
⇒ t = 0:0.01:15
⇒ xt = subs(x,t)
⇒ xtn = double(xt).
⇒ plot[t, xtn]
⇒ xlabel('t')
⇒ ylabel('x')
⇒ grid on
⇒ grid minor
⇒ grid right.

3. At steady state

$\underset{t\to\infty}{x} = \underset{\text{steady state}}{x} = 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = K\sin(t+a)$

$K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$.

NB: Coefficient of cost is $K\sin a$.
Coefficient of sint is $K\cos a$.

when squaring both sides.

$$K^2 \sin^2 a + K^2 \cos^2 a = 0.1^2 + 0.1^2$$

$$K^2 (\sin^2 a + \cos^2 a) = 0.02$$

$$K^2 = 0.02.$$

$$K = \sqrt{0.02}.$$

$$K = 0.144 = \frac{\sqrt{2}}{10}$$

$$\frac{K \sin a}{K \cos a} = \frac{0.1}{0.1} = 1$$

recall that $\sin/\cos = \tan$

$$\tan a = 1.$$

$$\tan^{-1}(1) = a.$$

$a = 45°$ or $\pi/4$ radian

$\Rightarrow$ steady state

$$= \frac{\sqrt{2}}{10} \sin(t + \pi/4)$$