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17/ENG06/070

MECHANICAL ENGINEERING

ASSIGNMENT 3

1a. $x(x-1)y + (3x-1)y + y = 0$

Applying leibnits - maclaurin method

$(x^2-x)y'' + (3x-1)y' + y = 0$

$(x^2-x)y^2 + (3x-1)y' + y = 0$

Find the nth derivative using leibnits theorem

let $(x^2-x)y^{(2)} = w_1$

$y^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \cdots$

$u = y^2 \qquad V = x^2 - x.$

$u^n = y^{(n+2)} \qquad V' = 2x - 1$

$u^{n-1} = y^{(n+1)} \qquad V'' = 2$

$u^{n-2} = y^n \qquad V''' = 0$

$$w_1^{(n)} = y^{(n+2)} \times (x^2-x) + ny^{(n+1)} \times (2x-1) + \frac{n(n-1)}{2!} y^n \times 2.$$

let $w_2 = (3x-1)y^{(1)}$

$u = y^{(1)} \qquad V = 3x-1$

$u^{(n)} = y^{(n+1)} \qquad V' = 3$

$u^{(n-1)} = y^n \qquad V'' = 0$

$$w_2^{(n)} = y^{(n+1)} \times (3x-1) + ny^{(n)} \times 3$$

let $w_3 = y$

$w_3^{(n)} = y^{(n)}$

$\therefore$ nth derivative

$$y^{(n+2)} = (x^2-x) + ny^{(n+1)} \times (2x-1) + n(n-1)y^{(n)} + y^{(n+1)} \times (3x-1) + ny^{(n)} \times 3 + y^n = 0$$

Assuming $x = 0$

$n(n-1)y^{(n)} + 3ny^{(n)} + y^{(n)} - ny^{(n+1)} - y^{(n+1)} = 0$

$(n^2+2n+1)y^{(n)} - (n+1)y^{(n+1)} = 0$

$$y^{(n+1)} = \frac{(n^2+2n+1)y^{(n)}}{(n+1)} = \frac{\cancel{(n+1)}(n+1)\ y^{(n)}}{\cancel{(n+1)}}$$

$\therefore \ y^{(n+1)} = (n+1)y^{(n)}$

when $n=0$; $(y^{(1)})_0 = (y^{(0)})_0$

$n=1$; $(y^{(2)})_0 = (2)(y')_0 = (2y)_0$

$n=2$; $(y^3)_0 = \cancel{(3)(y^2)}\ \cancel{3\times 2 \times y^0}\ 3(y^{(2)})_0 = 6(y^{(0)})_0$

$n=3; \; (y^{(4)})_0 = 4(y^{(3)})_0 = 4 \times 6(y^{(1)}) = (24 y^{0})$

$n=4; \; (y^{5}) = 5(y^{4})_0 = 5 \times 24 (y^{(0)}) = (120 y^{0})_0$

$n=5; \; (y^{6}) = 6(y^{5})_0 = 6 \times 120 (y^{(0)}) = (720 y^{0})_0$

$n=6; \; (y^{7}) = 7(y^{6})_0 = 7 \times 720 (y^{(0)}) \; (5040 \, y^{0})_0$

Maclaurin Series

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{2})_0 + \frac{x^3}{3!}(y^{3})_0 + \cdots$$

$$y = (y^{0})_0 + x(y^{0})_0 + \frac{x^2}{2!}(2(y^{0})_0) + \frac{x^3}{3!}(6 y^{(0)})_0 + \frac{x^4}{4!}(24(y^{(0)})_0) + \frac{x^5}{5!}(120(y^{0})_0) +$$

$$\frac{x^6}{6!}(720(y^{0})_0) + \frac{x^7}{7!}(5040(y^{0})_0 + \cdots$$

$$y = (y^{0})_0 + x(y^{(0)})_0 + x^2(y^{0})_0 + x^3(y^{0})_0 + x^4(y^{0})_0 + x^5(y^{0})_0 + x^6(y^{0})$$
$$(y^{0})_0 + \cdots$$

a) Power series, $y = (y^{(0)})_0 (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots)$

Since $y(0) = 0.0005m$ and $y'(0) = 0.0005$,

power series; $y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots)$.

b. when $x = 5m$

$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots)$

$= 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7 + \cdots)$

$y = 48.828m.$

when $x = 8m.$

$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots)$

$= 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$= 1198.3735m$

when $x = 10m$

$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots)$

$= 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7 + \cdots)$

$\therefore = 5555.5555m$

C. command window

clear

clc

Close all

x = 0:0.01:10

```
y=0.0005*(1+x+x.^2+x.^3+x.^4+x.^5+x.^6+x.^7).
yn= subs(y).
Plot (x,yn)
x label ('n')
y label ('Deflection')
axis tight
grid on.
grid minor.
```