

Differential Equations

17/10/2021

COMPUTER EXERCISE

(1) $x(x-1)y'' + (5x-1)y' + y = 0$

For the nth derivative,

$$x(x-1)y^{(n+2)} + (2x-1)y^{(n+1)} + n(n-1)y^{(n)} + (5x-1)y^{(n+1)} + 5ny^{(n)} + y^{(n)} = 0$$

at $x=0$;

$$(y^{(n+1)})_0 (5n-1) + (y^{(n)})_0 (n^2 + 2n + 1) = 0$$

$$(y^{(n+1)})_0 = (n+1)(y^{(n)})_0$$

at $n=0$,

$$y'_0 = y_0$$

at $n=1$,

$$y^{(2)}_0 = (1+1)(y^{(1)})_0$$

$$y^{(2)}_0 = 2y'_0$$

$$y^{(3)}_0 = 3y^{(2)}_0 = 6y^{(1)}_0$$

$$\therefore y^{(3)}_0 = 6y^{(1)}_0$$

at $n=2$,

$$y^{(4)}_0 = (3+1)(y^{(3)})_0$$

$$y^{(4)}_0 = 4y^{(3)}_0 = 24y^{(1)}_0$$

$$\therefore y^{(4)}_0 = 24y^{(1)}_0$$

at $n=3$,

$$y^{(5)}_0 = (4+1)(y^{(4)})_0$$

$$y^{(5)}_0 = 5y^{(4)}_0 = 120y^{(1)}_0$$

$$\therefore y^{(5)}_0 = 120y^{(1)}_0$$

at $n=4$,

$$y^{(6)}_0 = (5+1)(y^{(5)})_0$$

$$y^{(6)}_0 = 6y^{(5)}_0 = 720y^{(1)}_0$$

$$y^{(6)}_0 = 720y^{(1)}_0$$

at $n=6$,

$$y^{(6)} = (6+1)(y^{(5)})$$

$$y^{(5)} = 7(y^{(4)}) \quad y^{(4)} = 720y^{(3)}$$

$$y^{(3)} = 7(120)y^{(2)} = 5040y^{(2)}$$

Using Maclaurin Series

$$y = y_0 + x(y_1)_0 + \frac{x^2}{2!}(y_2)_0 + \frac{x^3}{3!}(y_3)_0 + \frac{x^4}{4!}(y_4)_0 + \frac{x^5}{5!}(y_5)_0 + \dots$$

$$\therefore y = y_0 + x(y_1)_0 + \frac{x^2}{2!}(5040y_0) + \frac{x^3}{3!}(720y_0) + \frac{x^4}{4!}(720y_0) + \frac{x^5}{5!}(720y_0) + \dots$$

$$+ \frac{x^6}{6!}(720y_0) + \frac{x^7}{7!}(5040y_0) + \dots$$

$$= y_0 + xy_0 + \frac{x^2}{2}y_0 + \frac{x^3}{6}y_0 + \frac{x^4}{24}y_0 + \frac{x^5}{120}y_0 + \frac{x^6}{720}y_0 + \frac{x^7}{5040}y_0 + \dots$$

$$= y_0 + y_0 [x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \dots]$$

but $y_0 = 0.0005m$ and $y_0 = 0.0005$

(B) when $x = 5m$,

$$y = 0.0005m + 0.0005 [5 + 2.5 + 1.25 + 0.625 + 0.3125 + 0.15625 + 0.078125]m$$

$$= 0.0005m + 0.0005 [9.765625]m$$

$$= 48.828m$$

when $x = 8m$,

$$y = 0.0005m + 0.0005 [8 + 64 + 15.12 + 4.096 + 52.768 + 262.144 + 2097.152]m$$

$$= 0.0005m + 0.0005 [2596.744]m$$

$$= 1198.3725m$$

when $x = 10m$,

$$y = 0.0005m + 0.0005 [10 + 100 + 1666.666 + 10000 + 100000 + 1000000]$$

$$= 0.0005m + 0.0005 [1111100]$$

$$= 5555.5555m$$

② Command window

Clear

clc

Syms x

Syms y

$x = (0:2:6);$

$$y = 0.005 + 0.0005 \left[x + (x \cdot 1.2) + (x \cdot 1.3) + (x \cdot 1.4) + (x \cdot 1.5) + (x \cdot 1.6) + (x \cdot 1.7) \right]$$

Plot (x,y)

grid on

hold on

x label ('x')

y label ('Structural Deformation')