

Olutayo Ayodeji Samuel

171ENG 05 1034

Mechatronics

ENG 281

$$1) \quad x(x-1)y'' + (3x-1)y' + y = 0$$

Expanding the bracket

$$(x^2 - x)y'' + (3x-1)y' + y = 0$$

$$w_1 = (x^2 - x)y''$$

$$w_2 = (3x-1)y'$$

$$w_3 = y$$

Using Leibnitz theorem

$$u^n v + \frac{n u^{n-1} v'}{2!} + \frac{n(n-1) u^{n-2} v''}{2!} + \frac{n(n-1)(n-2) u^{n-3} v'''}{3!}$$

For  $w_1$

$$u = y^2 \quad v = x^2 - x$$

$$u^n = y^{n+2} \quad v' = 2x - 1$$

$$u^{n-1} = y^{n+1} \quad v'' = 2$$

$$u^{n-2} = y^n \quad v''' = 0$$

For  $w_2$

$$u = y \quad v = 3x - 1$$

$$u^n = y^n \quad v' = 3$$

$$y^{n-1} = y^n \quad v'' = 0$$

For  $w_3$

$$u = y \quad v = 1$$

$$u^n = y^n \quad v' = 0$$

$$W_1 = y^{(n+2)} \cdot (x^2 - x) + n \cdot y^{(n+1)} (2x - 1) + \frac{n(n-1)}{2!} y^n \cdot 2$$
$$= (x^2 - x) y^{(n+2)} + n(2x - 1) y^{(n+1)} + n(n-1) y^n$$

$$W_2 = y^{(n+1)} \cdot (3x - 1) + 3ny^n$$

$$W_3 = y^n$$

Summing all together

$$(x^2 - x)y^{(n+2)} + n(2x - 2)y^{(n+1)} + (n^2 - n)y^n + y^{(n+1)}(3x - 1) + 3ny^n + y^n = 0$$

assuming  $x=0$

$$(0-0)y^{(n+2)} + (2n(0) - n)y^{(n+1)} + (n^2 - n)y^n + (3(0) - 1)y^{(n+1)} + 3ny^n + y^n = 0$$

$$= -ny^{(n+1)} + (n^2 - n)y^n - y^{(n+1)} + 3ny^n + y^n = 0$$

Collecting like terms

$$y^{(n+1)}(-n - 1) + y^n(n^2 - n + 3n + 1) = 0$$

$$= -y^{(n+1)}(n+1) + y^n(n^2 + 2n + 1) = 0$$

$$(n+1)y^{(n+1)} = (n^2 + 2n + 1)y^n \quad [(n^2 + 2n + 1) = (n+1)(n+1)]$$

$$(n+1)y^{(n+1)} = (n+1)(n+1)y^n$$

Divide both sides by  $(n+1)$

$$y^{(n+1)} = (n+1)y^n \quad \text{Recurrence relation}$$

$$(y^{(n+1)})_0 = y^n (n+1)$$

$$(y^0)_0 = 0.0005$$

$$(y^1)_0 = 0.0005$$

When  $n=0$

$$\mathcal{L}\{y^{(0+1)}\}_0 = (0+1)(y^0)_0$$

$$\mathcal{L}\{y^{(1)}\}_0 = 1(y^0)_0$$

$$n=1; \quad \mathcal{L}\{y^{(1+1)}\}_0 = (1+1)(y^1)_0$$

$$\mathcal{L}\{y^{(2)}\}_0 = 2(y^1)_0$$

$$n=2; \quad (y^3)_0 = (2+1)y^2$$

$$= 3(y^2)_0 = 3\mathcal{L}\{2(y^1)_0\} = 6(y^1)_0$$

$$n=3; \quad (y^4)_0 = (3+1)y^3$$

$$4(y^3)_0 = 4(6(y^1)_0) = 24(y^1)_0$$

$$n=4; \quad (y^5)_0 = (4+1)y^4$$

$$5(y^4)_0 = 5(24(y^1)_0) = 120(y^1)_0$$

$$n=5; \quad (y^6)_0 = (5+1)y^5$$

$$= 6(y^5)_0 = 6(120(y^1)_0) = 720(y^1)_0$$

$$n=6; (y^2)_0 = (6+1)y^0$$

$$= 7y^6 = 7(720 y^1)_0 = 5040(y^1)$$

Using Maclaurin series

$$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$$

$$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(24^1)_0 + \frac{x^3}{3!}(64^1)_0 + \frac{x^4}{4!}(24y^1)_0 + \frac{x^5}{5!}(120y^1)_0 + \frac{x^6}{6!}(720y^1)_0 + \frac{x^7}{7!}(5040y^1)_0$$

$$y = y^0(1+x) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)y^1$$

$$y = 0.0005(1+x) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)0.0005$$

ii) Estimate the approximate deformation when  $x = 5, 8$  and  $10m$  when  $\alpha = 5m$

$$y = y^0(1+5) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)0.0005$$

$$= 0.0005(6) + (25 + 125 + 625 + 3125 + 15625 + 78125)0.0005$$

$$y = 48.828m$$

when  $x = 8m$

$$y = y^0(1+8) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)y^1$$

$$y = 0.0005(1+8) + (64 + 512 + 4096 + 32768 + 262080 + 209715)0.0005$$

$$= 1198.3725m$$

when  $x = 10m$

$$y = y^0(1+10) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)y^1$$

$$= 0.0005(11) + (100 + 1000 + 10000 + 100000 + 1000000 + 10000000)0.0005$$

$$= 5555.56m$$

Matlab mfile

Command Window

Clear

clc

Close all

$x = 0 : 0.001 : 10$

$Y = (0.00005 * (1+x)) + (0.00005 * x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$Y_n = \text{subs}(Y)$

plot(x, Y\_n)

x label ('m')

y label ('Deflection')

axis tight

grid on

grid minor

(A) (deflection)

