

ENGINEERING MATHS

1)  $x(x-1)y'' + (3x-1)y' + y = 0$

$(x^2-x)y'' + (3x-1)y' + y = 0$  Expanding bracket

We can then assume

$(x^2-x)y'' = w_1, (3x-1)y' = w_2, y = w_3$

Using Leibnitz theorem  $= U^n V + nU^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} + \dots$

$w_1$	$w_2$	$w_3$
let $U = y^n, V = x^2 - x$	let $U = y^1, V = 3x - 1$	let $U = y, V = 1$
$U^n = y^{n+2}, V' = 2x - 1$	$U^n = y^{n+1}, V' = 3$	$U^n = y^n, V' = 0$
$U^{n-1} = y^{n+1}, V'' = 2$	$U^{n-1} = y^n, V'' = 0$	
$U^{n-2} = y^n, V''' = 0$		

$w_1 = (y^{n+2} \cdot (x^2-x)) + n(y^{n+1})(2x-1) + \frac{n(n-1)}{2!} y^{n-2}$

$w_2 = (y^{n+1} \cdot (3x-1)) + n y^n \cdot 3 + 0$

$w_3 = y^n$

$\therefore w_1 + w_2 + w_3 = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + n(n-1)y^n + y^n + (3x-1) + ny^n \cdot 3 + y^n = 0$

$= y^{n+2}(x^2-x) + y^{n+1}(2x-n) + y^{n+1}(3x-1) + n(n-1)y^n + ny^n \cdot 3 + y^n = 0$

assuming  $x=0$   
 $= y^{n+2}(0^2-0) + y^{n+1}(2(0)-n) + y^{n+1}(3(0)-1) + n(n-1)y^n + ny^n \cdot 3 + y^n = 0$

$= -n y^{n+1} - y^{n+1} + y^n (n^2-n) + 3n y^n + y^n = 0$

~~$= -y^{n+1}(-n-1) + y^n(n^2-n) + 3n y^n + y^n = 0$~~

$= y^{n+1}(-n-1) + y^n(n^2-n+3n-1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2+2n-1) = 0$

$y^{n+1}(n+1) = y^n(n^2+2n-1)$

$y^{n+1} = y^n(n+1)$  re-occurrence relation

$(y^{n+1})_0 = y^n(n+1)_0$

$(y^{11})_0 = 0.0005$

$(y^1)_0 = 0.0005$

when  $n=0$  :  $(y^{(n+1)})_0 = (y^{(11)})_0 (0+1)$

$$(y^1)_0 = (y^0)_0 = 1$$

$$\text{When } n=1: (y^{(1+1)})_0 = (y^1)_0 (1+1)$$

$$(y^2)_0 = (y^1)_0 \cdot 2 = 2(y^1)_0$$

$$\text{When } n=2: (y^{(2+1)})_0 = (y^2)_0 (2+1)$$

$$(y^3)_0 = 3(y^2)_0 = 6(y^1)_0$$

$$\text{When } n=3: (y^{(3+1)})_0 = (y^3)_0 (3+1)$$

$$(y^4)_0 = (y^3)_0 \cdot 4 = 24(y^1)_0$$

$$\text{When } n=4: (y^5)_0 = (y^4)_0 \cdot 5 = 120(y^1)_0$$

$$\text{When } n=5: (y^6)_0 = (y^5)_0 \cdot 6 = 720(y^1)_0$$

$$\text{When } n=6: (y^7)_0 = (y^6)_0 \cdot 7 = 5040(y^1)_0$$

Using Maclaurin Series

$$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$$

$$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!} \cdot 2(y^1)_0 + \frac{x^3}{3!} \cdot 6(y^1)_0 + \frac{x^4}{4!} \cdot 24(y^1)_0 + \frac{x^5}{5!} \cdot 120(y^1)_0 \\ + \frac{x^6}{6!} \cdot 720(y^1)_0 + \frac{x^7}{7!} \cdot 5040(y^1)_0$$

$$y = (y^0)_0 + (x+1) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(y^1)_0$$

$$\therefore y = 0.0005(x+1) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) \cdot 0.0005$$

ii) When  $x = 5m$

$$y = 0.0005(5+1) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7) \cdot 0.0005$$

$$\therefore y = 48.828m$$

When  $x = 8m$

$$y = 0.0005(8+1) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7) \cdot 0.0005$$

$$\therefore y = 4198.373m$$

When  $x = 10m$

$$y = 0.0005(10+1) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7) \cdot 0.0005$$

$$\therefore y = 5555.56m$$

iii) MATLAB m file

Command window

clear

clc

Close all

$$x = 0.001 : 10$$

$$y = (0.0005 * (x+1)) + ((x^2 + x^3 + x^4 + x^5 + x^6 + x^7) * (0.0005))$$

$$y_n = \text{subs}(Y)$$

plot(x, y\_n)

axis tight

grid on

grid minor

x label ('m')

y label ('Deflection')

Sketch

Graph of Structural Element Against  $x$

