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1) $y = 3e^{2x}$ and $y = 3e^{-x}$ at the points $x=1$ and $x=2$. Find the area bounded by the curves.

Soln

Area between 2 curves

$$A = \int_a^b f(x) - g(x) dx \quad \text{where } f(x) = 3e^{2x} \text{ and } g(x) = 3e^{-x}$$

$$a=1, b=2$$

$$A = \int_1^2 3e^{2x} - (3e^{-x}) dx$$

$$A = \int_1^2 3e^{2x} - 3e^{-x} = 3 \int_1^2 e^{2x} - e^{-x}$$

$$A = 3 \left[\frac{e^{2x}}{2} + e^{-x} \right]_1^2$$

$$A = 3 \left[\left(\frac{e^{2(2)}}{2} + e^{-2} \right) - \left(\frac{e^{2(1)}}{2} + e^{-1} \right) \right]$$

$$A = 3 \left[(27.2 + 0.135) - 4.06 \right]$$

$$A = 3(23.275)$$

$$= 69.823 \text{ units}^2 \approx 70 \text{ units}^2$$

$$2) y = 2 \sin \frac{\pi}{10} t \quad x = 2t + 2t - 2 \cos \frac{\pi}{10} t$$

$$A = \int_a^b y \, dx \quad \text{let } \frac{\pi}{10} = u$$

$$x = 2 + 2t - 2 \cos \frac{\pi}{10} t$$

$$\frac{dx}{dt} = 2 + 2 \sin ut$$

$$dx = 2 + 2 \sin ut \, dt$$

$$A = \int_0^{10} 2 \sin ut (2 + 2 \sin ut) \, dt$$

$$= 4 \int_0^{10} \sin ut + 4 \sin^2 ut \, dt$$

$$\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$$

$$= 4 \int_0^{10} \sin ut + \frac{4t}{2} (1 - \cos 2ut) \, dt$$

$$= 4 \int_0^{10} \sin ut + \frac{4t}{2} - \frac{4t}{4u} \cos 2ut \, dt$$

$$C = \frac{\pi}{10}$$

$$= 4 \left[\frac{4t}{20} - \frac{10}{\pi} \cos \frac{\pi}{10} t - \frac{1}{4} \sin \frac{\pi}{5} t \right]_0^{10}$$

$$= \left[\frac{4t}{6} - \frac{40}{\pi} \cos \frac{\pi t}{10} - \frac{\sin \pi t}{5} \right]_0^{10}$$

$$= \left(\frac{4(10)}{5} - \frac{\sin \pi(10)}{5} - \frac{40 \cos \frac{\pi(10)}{10}}{\pi} \right) - \left(\frac{4(0)}{6} - \frac{\sin \pi(0)}{5} - \frac{40 \cos \frac{\pi(0)}{10}}{\pi} \right)$$

$$= 2\pi + \frac{40}{\pi} + \frac{40}{\pi} = 31.75 \text{ square units}$$