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18/ENG01/0??
CHEMICAL ENGINEERING

1) $y = 3e^{3x}$ and $y = 3e^{-x}$ at the point $x = 1$ and $x = 2$ find the area bounded bounded by the Curves Soln $A = \int_1^2 3e^{2x}\,dx - \int_1^2 3e^{-x}\,dx$

$$\text{Area} = \left[\frac{3}{2}e^{2x} + c\right]_1^2 - \left[-3e^{-x} + c\right]_1^2$$

$$= \left[\frac{3}{2}(e^2 - e^4) + c - c\right] - \left[-3e^{-2} + 3e^{-1} + c - c\right]$$

$$= [70.81] - [0.698]$$

$$A = 70.112 \text{ units}^2$$

2) $y = 2\sin\left(\frac{\pi t}{10}\right)$, $x = 2 + 2t - 2\cos\left(\frac{\pi t}{10}\right)$ at ordinates $t_2 = 10$ and $t_1 = 0$

Soln

$$\frac{dx}{dt} = 2 - 2\cdot\left(-\sin\frac{\pi t}{10}\right) \times \frac{\pi}{10}$$

$$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi t}{10}$$

$$dx = 2 + \frac{\pi}{5}\sin\left(\frac{\pi t}{10}\right)dt$$

Area bounded by the parametric equations $(A) = \int_{x_1}^{x_2} y\,dx$

$$A = \int_{x_1}^{x_2} 2\sin\left(\frac{\pi t}{10}\right)dx$$

$$A = \int_{t_1}^{t_2} 2\sin\frac{\pi t}{10}\left(2 + \frac{\pi}{5}\sin\left(\frac{\pi t}{10}\right)\right)dt$$

$$A = \int_0^{10} 4\sin\left(\frac{\pi t}{10}\right) + \frac{2\pi}{5}\sin^2\left(\frac{\pi t}{10}\right)dt$$

$$A = \int_0^{10} 4\sin\left(\frac{\pi t}{10}\right)dt + \int_0^{10}\frac{2\pi}{5}\sin^2\left(\frac{\pi t}{10}\right)dt$$

But, $\cos^2 x - \sin^2 x = \cos 2x$

$$1 - \sin^2 x - \sin^2 x = \cos x$$

$$1 - 2\sin^2 x = \cos 2x$$

$$\Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\therefore \int \sin^2 x\,dx = \int \frac{1 - \cos 2x\,dx}{2} = \frac{1}{2}\int 1 - \cos 2x\,dx$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x + c\right]$$

$$\therefore A = \left[4 \times \frac{10}{\pi} \times -\cos\left(\frac{\pi t}{10}\right) + c\right]_0^{10} + \left[\frac{2\pi}{5} \times \frac{10}{\pi} \times \frac{1}{2}\left(\frac{\pi t}{10} - \frac{1}{2}\sin\left(\frac{2\pi t}{10}\right) + c\right)\right]$$

$$A = \left[-\frac{40}{\pi}\cos\left(\frac{\pi t}{10}\right)+c\right]_0^{10} + \left[2\left(\frac{\pi t}{10} - \frac{1}{2}\sin\left(2\times\frac{\pi t}{10}\right)\right)+c\right]_0^{10}$$ 1)

$$A = \left[\frac{-40}{\pi}\cos\left(\frac{\pi\times 10}{10}\right)+c+\frac{40}{\pi}\left(\cos\left(\frac{\pi\times 0}{10}\right)\right)-c\right] + \left[2\left(\frac{\pi\times 10}{10}-\frac{1}{2}\sin\left(\frac{2\times\pi\times 10}{10}\right)\right)+c-2\left(\frac{\pi\times 0}{10}+\frac{1}{2}\sin\left(\frac{2\times\pi\times 0}{10}\right)\right)-c\right]$$

$$A = \left[\frac{-40}{\pi}\cos\pi \;\; \frac{40}{\pi}\cos 0\right] + \left[2\pi - \frac{1}{2}\sin(2x) + \frac{1}{2}\sin(0)\right]$$

$$A = \left[\frac{-40}{\pi}\times -1) + \frac{40}{\pi}\right] + \left[2\pi - 0 + 0\right]$$

$$A = \frac{80}{\pi} + 2\pi$$

$$A = 31.75 \text{ units}^2$$

2)