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17/ENG06/072
MECHANICAL ENG
ENG 381

1) The model for the deformation (y) of a structural element is represented by the expression given in Equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0.$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin Method

a) Obtain the power series solution of the model up to and including the term in x^3,

b) estimate the approximate deformation when $x = 5$, 8 and 10m, and

c) with the aid of a MATLAB m-file program, plot the response of the structural element for $0 \leq x \leq 10m$.

Solu.

$$x(x-1)y'' + (3x-1)y' + y = 0.$$

$$x^2y'' - xy'' + 3xy' - y' + y = 0.$$

~~$x^2y - 2xy = 0$~~

Let $w_1 = x^2y$.

$w_2 = -2xy$.

$u = y \quad v = x^2$

$u^n = y^n \quad v^{(1)} = 2x$

$u^{n-1} = y^{n-1} \quad v^{(2)} = 2$

$u^{n-2} = y^n \quad v^{(3)} = 0$

$w_2 = -2xy$

$u = y$

$u^n = y^n \quad v^n = 0$

For w_1

$+y^{(n+2)} \cdot (x^2+x) + n \cdot y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2!} \cdot y^n \cdot 2 + \dots$

$w_1 = (x^2+x)y^{(n+2)} + n(2x-1)y^{(n+1)} + n(n-1)y^n$

For w_2.

$w_2 = y^{(n+1)} \cdot 2x-1 + n \cdot y^{(n)} \cdot 3 + \dots$

$= (3x-1)y^{(n+1)} + 3ny^{(n)} \dots$

For w_3.

$= y^n \cdot 1$

$= y^n$.

Adding $w_1 + w_2 + w_3$.

$(x^2+x)y^{(n+2)} + n(2x-1)y^{(n+1)} + n(n-1)y^n + (3x-1)y^{(n+1)} + 3ny^{(n)} + y^n = 0$.

Substitute $x = 0$.

$y^n + 0 \cdot y^{(n+2)} + n(-1)y^{(n+1)} + n(n-1)y^n + 0 \cdot y^{(n+1)} + 3ny^{(n)} = 0$

$y^n + n(n-1)y^n + 3ny^{(n)} = 0$

$y^n(1 + n(n-1) + 3n) = 0$

$-y^{(n+1)} + y^n - ny^{(n+1)} + n(n-1)y^n + 3ny^{(n)} = 0$

$y^{(n+1)}(-1-n) + y^n(1 + n(n-1) + 3n) = 0$.

$y^{(n+1)} = \frac{-y^n(1 + n^2 - n + 3n)}{-(1+n)}$

$y^{(n+1)} = \frac{y^n(1 + n^2 + 2n)}{(1+n)}$

$(y^{(n+1)})_0 = \frac{y^n(n+1)(n+1)}{(n+1)}$

$(y^{(n+1)})_0 = y^n(n+1)$

$n=0, (y^{(1)})_0 = (y^0)_0$

when $n=1$, $(y^{(2)})_0 = (y^1)_0 2 = 2(y^0)_0$ or $2y^{(0)})_0$

when $n=2$, $(y^{(3)})_0 = 3(y^{(2)})_0 = 3 \times 2(y^{(1)})_0 = 3(y^{(1)})_0$

when $n=3$, $(y^{(4)})_0 = 4(y^{(3)})_0 = 4 \times 3 \times 2(y^{(1)})_0 = 4!(y^{(1)})_0$

when $n=4$, $(y^{(5)})_0 = 5(y^{(4)})_0 = 5 \times 4!(y^{(1)})_0 = 5!(y^{(1)})_0$

when $n=5$, $(y^{(6)})_0 = 6(y^{(5)})_0 = 6 \times 5!(y^{(1)})_0 = 6!(y^{(1)})_0$

" $n=6$, $(y^{(7)})_0 = 7(y^{(6)})_0 = 7 \times 6!(y^{(1)})_0 = 7!(y^{(1)})_0$

when $n=7$, $(y^{(8)})_0 = 8(y^{(7)})_0 = 8\times 7!\,(y^{(6)})_0 = 8!\,(y^{(6)})_0$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}\cdot 2(y^{(2)})_0 + \frac{x^3}{3!}\cdot 3!\,(y^{(3)})_0 + \frac{x^4}{4!}\cdot$

$+ \frac{x^5}{5!}\cdot 5!\,(y^{(5)})_0 + \frac{x^6}{6!}\cdot 6!\,(y^{(6)})_0 + \frac{x^7}{7!}\cdot 7!\,(y^{(7)})_0 + \frac{x^8}{8!}\cdot$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + x^2(y^{(2)})_0 + x^3\cdot(y^{(3)})_0 + x^4(y^{(4)})_0 + x^5(y^{(5)})_0$

$+ x^6(y^{(6)})_0 + x^7(y^{(7)})_0 + x^8(y^{(8)})_0$

$y = (y^{(0)})_0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$ ~~$+ x^8 x^8$~~

b) when $x = 5$, 8, and 10m.

$(y^{(0)})_0 = 0.0005$m

$(y'^{(0)})_0 = 0.0005$

$y = (y^{(0)})_0(1+x) + (y^{(1)})_0(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)x$

c) when $x = 5, 8, 10$m.

$(y^{(0)})_0 = 0.0005$m.

$(y'^{(0)})_0 = 0.0005$

when $x = 5$

$y = 0.0005\text{m}(1+5) + (0.005)(5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$y = 0.0005\text{m} + 0.0025\text{m} + 48.825\text{m}$

$y = 48.825\text{m} + 0.0005\text{m} + 0.0025\text{m}$

$y = 48.828$m.

when $x = 8$m.

$y = 0.0005\text{m}(1+8) + 0.0005(8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$y = (1198.37 + 0.0005 + 4\times 10^{-3})$ m

$y = 1198.37$m.

when $x = 10$m.

$y = 0.0005\text{m}(1+10) + 0.0005(10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$y = 5.5\times 10^{-3} + 0.0005(11111100)$

$y = 5.5\times 10^{-3} + 5555.55$

$y = 5555.556$m.

c. Commandwindow

```
- Clear all
- clc.
- Close all.
- syms x.
x = 0:0.01:10
y = 0.00005*(1+x) + (((x^2)+(x^3)+(x^4)+(x^5)+(x^6)+
    (x^7))*(0.00005)).
plot(x,y)
- grid minor
- grid ~~major~~ on
- xlabel('time')
- ylabel('Decay')
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