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1) $y=3e^{2x}$ and $y=3e^{-x}$ at the point $x=1$ and $x=2$. Find the area bounded by the curves.

Solution

Area between 2 curves

$A = \int_a^b f(x) - g(x)\,dx$, where $f(x) = 3e^{2x}$ and $g(x) = 3e^{-x}$

$a=1$, $b=2$

$A = \int_1^2 3e^{2x} - (3e^{-x})dx$

$A = \int_1^2 3e^{2x} - 3e^{-x} = 3\int_1^2 e^{2x} - e^{-x}$

$= 3\left[\left(\frac{e^{2x}}{2} + e^{-2}\right) - \left(\frac{e^{2(1)}}{2} + e^{-1}\right)\right]$

$A = 3\left[(27.2 + 0.135) - 4.06\right]$

$A = 3(23.275)$

$= 69.823$ units^2 $\simeq 70$ units^2

2) $y = 2\sin \pi/_{10} t \qquad x = 2 + 2t - 2\cos \pi/_{10} t$

$$A = \int_a^b y\,dx \qquad \text{let } \pi/_{10} = u$$

$$x = 2 + 2t - 2\cos \pi/_{10} t$$

$$\frac{dx}{dt} = 2 + 2u\sin ut$$

$$dx = 2 + 2u\sin ut\,dt$$

$$A = \int_0^{10} 2\sin ut\,(2 + 2u\sin ut)\,dt$$

$$= 4\int_0^{10} \sin ut + u\sin^2 ut\,dt$$

$$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$$

$$= 4\int_0^{10} \sin ut + \frac{u}{2}(1 - \cos 2ut)\Big]\,dt$$

$$= 4\left[-\frac{\cos ut}{u} + \frac{ut}{2} - \frac{u}{4u}\sin 2ut\right]_0^{10}$$

$$C = \pi/_{10}$$

$$= 4\left[\frac{\pi t}{20} - \frac{10}{\pi}\cos\frac{\pi}{10}t - \frac{1}{4}\sin\frac{\pi}{5}t\right]_0^{10}$$

$$= \left[\frac{\pi t}{6} - \frac{40}{\pi}\frac{\cos\pi t}{10} - \frac{\sin\pi t}{5}\right]_0^{10}$$

$$= \left(\frac{\pi(10)}{5} - \frac{\sin\pi(10)}{5} - \frac{40}{\pi}\cos\frac{\pi(10)}{10}\right) - \left(\frac{\pi(0)}{6} - \frac{\sin\pi(0)}{5} - \frac{40}{\pi}\frac{\cos\pi(0)}{10}\right)$$

$$= 2\pi + 40/\pi + 40/\pi = 31.75 \text{ square units}$$