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Assignment

The model for the deformation (y) of a structural element is represented by the expression given in equation (1) $x(x-1)y'' + (3x-1)y' + y = 0$

Given that $y(0) = 0.0005$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin Method

a) obtain the power series solution of the model up to and including the term in x^7 ,

b) estimate the appropriate deformation when $x = 5, 8$ and 10

c) with the aid of a Matlab mfile program, plot the response of the structural element for $0 \leq x \leq 10m$.

Solution:

Using Leibnitz method,

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + \frac{n(n-1)}{2!} 2y^n + (3x-1)y^{n+1} + n(3)y^n + y^n = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2-n+3n+1)y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

when $x=0$

$$(2n(0)-n+3(0)-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$\div (n+1)y^{n+1} = -(n^2+2n+1)y^n$$

$$y^{n+1} = \frac{(n^2+2n+1)y^n}{(n+1)}$$

$$(y^{n+1})_0 = \frac{(n+1)(y^n)_0}{(n+1)}$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

$$\text{at } n=0; y^1 = 1y^0$$

$$n=1; y^2 = 2(y^1)_0$$

$$n=2; y^3 = 3y^2 = 3(2)y' = 6(y')_0$$

$$n=3; y^4 = 4y^3 = 4(3)(2)y' = 24(y')_0$$

$$n=4; y^5 = 5y^4 = 5(4)(3)(2)y' = 5!(y')_0$$

$$n=5; y^6 = 6y^5 = 6(5)(4)(3)(2)y' = 6!(y')_0$$

$$n=6; y^7 = 7y^6 = 7(6)(5)(4)(3)(2)y' = 7!(y')_0$$

Maclaurin Series

$$y = y_0 + \frac{x(y'_0)}{1!} + \frac{x^2(y''_0)}{2!} + \frac{x^3(y'''_0)}{3!} + \frac{x^4(y^{(4)}_0)}{4!} + \frac{x^5(y^{(5)}_0)}{5!} + \frac{x^6(y^{(6)}_0)}{6!} + \frac{x^7(y^{(7)}_0)}{7!}$$

$$= y_0 + x(y'_0) + \frac{x^2(2y'_0)}{2!} + \frac{x^3(3!y'_0)}{3!} + \frac{x^4(4!y'_0)}{4!} + \frac{x^5(5!y'_0)}{5!} + \frac{x^6(6!y'_0)}{6!} + \frac{x^7(7!y'_0)}{7!}$$

$$= y_0 + x(y'_0) + x^2(y'_0) + x^3(y'_0) + x^4(y'_0) + x^5(y'_0) + x^6(y'_0) + x^7(y'_0)$$

$$= y_0 + y'_0(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Recall

$$y' = y^0$$

$$y_0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

$$y' = 0.0005 \text{ and } y^0 = 0.0005, \text{ when } x=5$$

$$y^5 = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$$

$$= 0.0005(97656)$$

$$= 48.828$$

$$\approx 49$$

$$\text{When } x=8; y_0 = 0.0005$$

$$y_8 = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$$

$$= 0.0005(2396745)$$

$$= 1198.3725$$

$$\approx 1198$$

$$x = 10$$

$$y_{10} = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$$

$$= 0.0005(11111111)$$

$$= 5555.5555$$

$$\approx 5556$$

Command Window

clc

clear all

close all

syms x y

x = 0:0.1:10

y = (0.0005)*(1+x+x²+x³+x⁴+x⁵+x⁶+x⁷)

Yn = subs(y)

Ynn = double(Yn)

Plot(x, Ynn)

x label('x')

y label('T')

grid on

grid minor

axis tight

