

1) $y = 3e^{2x}$

ordinate of $x = -1$ and $x = 2$

Area = $\int_a^b y_a dx - \int_a^b y_b dx$

$$\int_1^{23} 3e^{2x} = \left[\frac{3e^{2x}}{2} \right]_2^2 = \frac{3e^4 - 3e^2}{2}$$

= 70.81 square units

$y_b = 3e^{-x}$

$$\int_1^{23} 3e^{-x} = \left[-3e^{-x} - (-3e^{-1}) \right] = -3e^{-2} - (-3e^{-1}) = 0.698 \text{ square units}$$

Area bounded by the curves

= 70.81 - 0.698 = 70.116 square units

2) $y = 25 \sin \frac{\pi}{10} t$

$x = 2 + 2t - 2 \cos \frac{\pi}{10} t$

$t = 0$ and $t = 10$

Area = $\int_a^b y dx$

$\frac{dx}{dt} = 2 + \frac{\pi}{5} \sin \frac{\pi}{10} t$

$$A = \int_0^{10} 25 \sin \frac{\pi}{10} t \left[2 + \frac{\pi}{5} \sin \frac{\pi}{10} t \right] dt$$

$$= \int_0^{10} \left[45 \sin \frac{\pi}{10} t + 2\pi \sin^2 \left(\frac{\pi t}{10} \right) \right] dt$$

$$= \int_0^{10} 45 \sin \frac{\pi}{10} t + \int_0^{10} 2\pi \sin^2 \left(\frac{\pi t}{10} \right) dt$$

$$= \int_0^{10} -40005 \left(\frac{2t}{10} \right) + \frac{2\pi}{5} \times \pi t - 5 \sin \left(\frac{\pi t}{5} \right)$$

$$\begin{aligned}
&= \int_0^{10} \left[\frac{-40 \cos\left(\frac{\pi t}{10}\right)}{\pi} + \frac{\pi t - 5 \sin\left(\frac{\pi t}{5}\right)}{5} \right] \\
&= \int_0^{\pi} \left[\frac{-40 \cos\left(\frac{\pi(10)}{10} + \frac{\pi(10)}{10}\right) - 5 \sin\left(\frac{\pi(10)}{5}\right)}{5} - \frac{40 \cos\left(\frac{\pi(0)}{10} + \frac{\pi(0)}{10}\right) - 5 \sin\left(\frac{\pi(0)}{5}\right)}{5} \right] \\
&= \left[\frac{40 + 2\pi}{\pi} \right] - \left[\frac{-40 + 0}{\pi} \right] \\
&= \frac{40}{\pi} + 2\pi + \frac{40}{\pi} \\
&= \frac{80}{\pi} + 2\pi \\
&= 31.74 \text{ square units.}
\end{aligned}$$