



ENG381

ASSIGNMENT III

1. $x(x-1)y'' + (3x-1)y' + y = 0$

let $x(x-1)y'' = a_1$, $(3x-1)y' = a_2$, $y = a_3$

Considering a_1 ,

$u = y''$ $v = x(x-1)$

$u^n = y^{(n+2)}$ $v' = 2x-1$

$u^{n-1} = y^{(n+1)}$ $v'' = 2$

$u^{n-2} = y^n$ $v''' = 0$

a_2

$u = y'$ $v = (3x-1)$

$u^n = y^{(n+1)}$ $v' = 3$

$u^{n-1} = y^n$ $v'' = 0$

a_3

$u = y$ $v = 1$

$u^n = y^n$ $v' = 0$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \dots$$

$$a_1^n = y^{(n+2)} \cdot (x^2-x) + n y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$$

$$a_2^n = y^{(n+1)} \cdot (3x-1) + n y^n \cdot 3 + 0$$

$$a_3^n = y^n \cdot 1 + 0$$

Summation of a_1, a_2 and a_3

$$y^{(n+2)} \cdot (x^2-x) + n y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2} y^n \cdot 2 + y^{(n+1)} \cdot (3x-1) + y^n \cdot 1$$

$$n y^n \cdot 3 + y^n = 0$$

$$(x^2-x) y^{(n+2)} + (2x-1) n y^{(n+1)} + (3x-1) y^{(n+1)} + n(n-1) y^n + 3n y^n + y^n = 0$$

$$y^n = 0$$

$$(x^2 - x)y^{(n+2)} + (2xn - n)y^{(n+1)} + (3x - 1)y^{(n)} + (n^2 - n)y^n + 3ny^n + y^n = 0$$

$$(x^2 - x)y^{(n+2)} + (2xn + 3x - n - 1)y^{(n+1)} + (n^2 - n + 3n + 1)y^n = 0$$

$$(x^2 - x)y^{(n+2)} + (2xn + 3x - n - 1)y^{(n+1)} + (n^2 + 2n + 1)y^n = 0$$

$$(x - 1)xy^{(n+2)} + (2xn + 3x - n - 1)y^{(n+1)} + (n^2 + 2n + 1)y^n = 0$$

when $x = 0$

$$(0 - 1)0 y^{(n+2)} + (2(0)n + 3(0) - n - 1)y^{(n+1)} + (n^2 + 2n + 1)y^n = 0$$

$$(-n - 1)y^{(n+1)} + (n^2 + 2n + 1)y^n = 0$$

$$-n - (n+1)(y^{(n+1)})_0 + (n^2 + 2n + 1)(y^n)_0 = 0$$

$$[y^{(n+1)}]_0 = \frac{-(n+1)(y^{(n+1)})_0 + (n^2 + 2n + 1)(y^n)_0}{-(n+1)}$$

$$[y^{(n+1)}]_0 = \frac{(n+1)(n+1)(y^n)_0}{(n+1)}$$

$$(y^{(n+1)})_0 = (n+1)(y^n)_0$$

when $n = 0$

$$[y^{(0+1)}]_0 = (0+1)(y^{(0)})_0$$

$$(y^{(1)})_0 = 1(y^{(0)})_0$$

when $n = 1$

$$(y^{(1+1)})_0 = (1+1)(y^{(1)})_0$$

$$(y^{(2)})_0 = 2(y^{(1)})_0$$

when $n = 2$

$$(y^{(2+1)})_0 = (2+1)(y^{(2)})_0$$

$$(y^{(3)})_0 = 3(y^{(2)})_0 = 3(2)(y^{(1)})_0$$

$$(y^{(3)})_0 = 6(y^{(1)})_0$$

when $n = 3$

$$(y^{(3+1)})_0 = (3+1)(y^{(3)})_0$$

$$(y^{(4)})_0 = 4(y^{(3)})_0 = 4(6)(y^{(1)})_0$$

$$(y^{(4)})_0 = 24(y^{(1)})_0$$

when $n = 4$

$$(y^{(4+1)})_0 = (4+1)(y^{(4)})_0$$

$$(y^{(5)})_0 = 5(y^{(4)})_0 = 5(24)(y^{(1)})_0$$

$$(y^{(5)})_0 = 120(y^{(1)})_0$$

When $n=5$

$$(y^{(6)})_0 = (6+1)(y^{(5)})_0 \\ = 7(y^{(5)})_0 = 7(720)(y^{(4)})_0 \\ (y^{(6)})_0 = 5040(y^{(4)})_0$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(2y^{(1)})_0 + \frac{x^3}{3!} \cdot 6(y^{(1)})_0 + \frac{x^4}{4!} \cdot 24$$

$$(y^{(4)})_0 + \frac{x^5}{5!} \cdot 120(y^{(1)})_0 + \frac{x^6}{6!} \cdot 720(y^{(1)})_0 + \frac{x^7}{7!} \cdot 5040(y^{(1)})_0$$

When $n=5$

$$(y^{(6)})_0 = (5+1)(y^{(5)})_0 \\ (y^{(4)})_0 = 6(y^{(5)})_0 = 6(120)(y^{(1)})_0 \\ (y^{(6)})_0 = 720(y^{(1)})_0$$

When $n=6$

$$(y^{(7)})_0 = (6+1)(y^{(6)})_0 \\ = 7(y^{(6)})_0 = 7(720)(y^{(1)})_0 \\ (y^{(7)})_0 = 5040(y^{(1)})_0$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}$$

$$(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2}(2y^{(1)})_0 + \frac{x^3}{3!} \cdot 6(y^{(1)})_0 + \frac{x^4}{4!} \cdot 24$$

$$(y^{(1)})_0 + \frac{x^5}{5!} \cdot 120(y^{(1)})_0 + \frac{x^6}{6!} \cdot 720(y^{(1)})_0 + \frac{x^7}{7!} \cdot 5040(y^{(1)})_0$$

$$y = (1+x)(y^{(0)})_0 + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(y^{(1)})_0$$

$$y(0) = 0.0005 \text{ m}, y'(0) = 0.0005$$

$$y = (1+x)(0.0005 \text{ m}) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(0.0005)$$

when $x = 5 \text{ m}$, 8 m and 10 m

$$(b) \quad y = (1+5)(0.0005) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)(0.0005)$$

$$y = 3 \times 10^{-3} \text{ m} + 48.825 \text{ m}$$

$$y = 48.828$$

when $x = 8 \text{ m}$

$$y = (1+8)(0.0005) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)(0.0005)$$

$$y = 4.5 \times 10^{-3} + 2396736(0.0005)$$

$$y = 4.5 \times 10^{-3} + 1198$$

$$y = 1198 \text{ m}$$

when $x = 10 \text{ m}$

$$y = (1+10)(0.0005) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)(0.0005)$$

$$y = 5.5 \times 10^{-3} + 11111100(0.0005)$$

$$y = 5555.555 \text{ m}$$

$$y = 5556 \text{ m}$$

- 1 command window
- 2 clear
- 3 clc
- 4 close all
- 5 syms x
- 6 $x = ((1+x)^*(0.0005)) + ((x^2+x^3+x^4+x^5+x^6+x^7)*(0.0005))$
- 7 $t = 0:0.01:10$
- 8 $xt = \text{subs}(x,t)$
- 9 $xtn = \text{double}(xt)$
- 10 $\text{plot}(t, xtn)$
- 11 $\text{xlabel}('t')$
- 12 $\text{ylabel}('x')$
- 13 grid on
- 14 grid minor
- 15 axis tight

