

Ena 381

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$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$\text{Taking } x(x-1)y'' = a_1$$

$$(3x-1)y' = a_2$$

$$y = a_3$$

Considering  $a_1$

$$u = y'' \quad v = x(x-1)$$

$$u^n = y^{(n+2)} \quad v' = 2x-1$$

$$u^n = y^{(n+1)} \quad v'' = 2$$

$$u^n = y^n \quad v''' = 0$$

$a^2$

$$u = y' \quad v = (3x-1)$$

$$u = y^{(n+1)} \quad v' = 3$$

$$u^n = y^n \quad v'' = 0$$

$$y^n = u^n v + n u^{(n-1)} v' + n(n-1) u^{(n-2)} v'' + \frac{(n-1)(n-2)}{2!} u^{(n-3)} v''' + \dots$$

$$a_1^n = y^{(n+2)} \cdot (x^2-x) + n y^{(n+1)} \cdot (2x-1) + n \frac{(n-1)}{2!} y^n \cdot 2 + 0$$

$$a_2^n = y^{(n+1)} \cdot (3x-1) + n y^n \cdot 3 + 0$$

$$a_3^n = y^n \cdot 1 + 0$$

Summation of  $a_1, a_2$  and  $a_3$

$$y^{(n+2)} \cdot (x^2-x) + n y^{(n+1)} \cdot (2x-1) + n \frac{(n-1)}{2} y^n \cdot 2 + y^{(n+1)} \cdot (3x-1) + n y^n \cdot 3 + y^n = 0$$

$$(x^2-x)y^{(n+2)} + (2x-1)ny^{(n+1)} + (3x-1)y^{(n+1)} + n(n-1)y^n + 3ny^n + y^n = 0$$

$$(x^2-x)y^{(n+2)} + (2xn-n)y^{(n+1)} + (3x-1)y^{(n+1)} + (n^2-n)y^n + 3ny^n + y^n = 0$$

$$(x^2-x)y^{(n+2)} + (2xn+3x-n-1)y^{(n+1)} + (n^2-n+3n+1)y^n = 0$$

$$(x^2-x)y^{(n+2)} + (2xn+3x-n-1)y^{(n+1)} + (n^2+2n+1)y^n = 0$$

$$(x-1)x y^{(n+2)} + (2xn + 3x - n - 1)y^{(n+1)} + (n^2 + 2n + 1)y^n = 0$$

$$(0-1)0y \text{ when } x=0 + (2c0 + 3c0 - n - 1)y^{(n+1)} + (n^2 + 2n + 1)y^n = 0.$$

$$(-n-1)y^{(n+1)} + (n^2 + 2n + 1)y^n = 0$$

$$-(n+1)(y^{(n+1)})_0 + (n^2 + 2n + 1)(y^n)_0 = 0.$$

$$(y^{(n+1)})_0 = \frac{(n^2 + 2n + 1)(y^n)_0}{n+1}$$

$$[y^{(n+1)}]_0 = \frac{(n+1)(n+1)}{(n+1)} (y^n)_0$$

$$[y^{(n+1)}]_0 = (n+1) (y^n)_0$$

where  $n=0$

$$[y^{(n+1)}]_0 = (n+1) (y^{(n)})_0$$

$$(y^{(n+1)})_0 = 1 (y^{(0)})_0$$

where  $n=1$

$$(y^{(n+1)})_0 = (1+1) (y^{(1)})_0$$

$$(y^{(2)})_0 = 2 (y^{(1)})_0$$

where  $n=2$ .

$$(y^{(3)})_0 = (2+1) (y^{(2)})_0$$

$$(y^{(3)})_0 = (3) (y^{(2)})_0 = 3(2) (y^{(1)})_0$$

$$(y^{(3)})_0 = 6 (y^{(1)})_0$$

where  $n=3$

$$(y^{(4)})_0 = (3+1) (y^{(3)})_0$$

$$(y^{(4)})_0 = (4) (y^{(3)})_0 = 4(6) (y^{(2)})_0$$

$$[y^{(4)}]_0 = 24 (y^{(1)})_0$$

where  $n=4$ .

$$(y^{(5)})_0 = (4+1) (y^{(4)})_0$$

$$(y^{(5)})_0 = 5 (y^{(4)})_0 = 5(24) (y^{(1)})_0$$

$$[y^{(5)}]_0 = 120 (y^{(1)})_0$$

when  $n=5$ .

$$(y^{(6)})_0 = (5+1) (y^{(5)})_0$$

$$(y^{(6)})_0 = 6 (y^{(5)})_0 = 6(120) (y^{(1)})_0$$

$$(y^{(6)})_0 = 720 (y^{(1)})_0$$

when  $n=6$

$$(y^{(6)})_0 = (6+1) (y^{(5)})_0$$

$$= 7 (y^{(5)})_0 = 7(720) y^{(1)}_0$$

$$(y^{(6)})_0 = 5040 (y^{(1)})_0$$

$$a) y = (y^{(5)})_0 + x (y^{(4)})_0 + \frac{x^2}{2!} (y^{(3)})_0 + \frac{x^3}{3!} (y^{(2)})_0 + \frac{x^4}{4!} (y^{(1)})_0$$

$$+ \frac{x^5}{5!} (y^{(0)})_0 + \frac{x^6}{6!} (y^{(0)})_0 + \frac{x^7}{7!} (y^{(0)})_0$$

$$y = (y^{(0)})_0 + x (y^{(1)})_0 + \frac{x^2}{2!} (2 y^{(1)})_0 + \frac{x^3}{3!} \cdot 6 (y^{(1)})_0 + \frac{x^4}{4!}$$

$$\left[ (y^{(1)})_0 + \frac{x^5}{5!} \cdot 120 (y^{(1)})_0 + \frac{x^6}{6!} \cdot 720 (y^{(1)})_0 + \frac{x^7}{7!} \cdot 5040 (y^{(1)})_0 \right]$$

$$a) y = (1+x)(y^{(0)})_0 + (x^2+x^3+x^4+x^5+x^6+x^7) (y^{(1)})_0$$

$$y^{(0)} = 0.0005 \text{ m}, \quad y'(0) = 0.0005$$

$$y = (1+x)(0.0005 \text{ m}) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005)$$

when  $x = 5 \text{ m}, 8 \text{ m} \& 10 \text{ m}.$

B)

$$y = (1+5)(0.0005) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$$

$$y = 3 \times 10^{-3} \text{ m} + 97650 \text{ m} (0.0005 \text{ m})$$

$$y = 3 \times 10^{-3} \text{ m} + 48.825 \text{ m}$$

$$y = 48.828 \text{ m}$$

when  $x = 8 \text{ m}$

$$\tilde{y} = (1+8)(0.0005) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$$

$$y = 4.5 \times 10^{-3} + 2396736 (0.0005)$$

$$y = 4.5 \times 10^{-3} + 1198$$

$$y = 1198 \text{ m}$$

when  $x = 10 \text{ m}$

$$y = (1+10)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$$

$$y = 5.5 \times 10^{-3} + 1111100 (0.0005)$$

$$y = 5555.5555 \text{ m}$$

$$y = 5556 \text{ m} //$$

# MATLAB

## • Command window

Clear

clc

Close all.

Syms x

$x = (1+x)^{(0.0005)} + (x^2+x^3+x^4+x^5+x^6+x^7)^{(0.0005)}$

t = 0:0.01:10.

xt = subs(x,t)

xtn = double(xt).

Plot = plot(t, xtn)

xlabel('t')

ylabel('x')

grid on

grid minor

axis tight

Figure 1: Axis title

