Assignment

The model for the deformation (y) of a structural element is represented by the expression given in equation (1) $x(x-1)y'' + (3x-1)y' + y = 0$

Given that $y(0) = 0.0005$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin Method

a) obtain the power series solution of the model up to and including the term in x^7;

b) estimate the appropriate deformation when $x = 5, 8$ and 10

c) with the aid of a Matlab mfile program, plot the response of the structural element for $0 \le x \le 10m$.

Solution:

Using leibnitz method,

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + \frac{n(n-1)}{2!}2y^n + (3x-1)y^{n+1} + n(3)y^n + y^n = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2-n+3n+1)y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

when $x = 0$

$$(2n(0)-n+3(0)-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$-(n+1)y^{n+1} = -(n^2+2n+1)y^n$$

$$y^{n+1} = \frac{(n^2+2n+1)}{(n+1)}y^n$$

$$(y^{n+1})_0 = \frac{(n+1)\cancel{(n+1)}}{\cancel{(n+1)}}(y^n)_0$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

$n=0$; $y' = 1y^0$

$n=1$; $y^2 = 2(y')_0$

$n=2;\ y^3 = 3y^2 = 3(2)y' = 6(y')_0$
$n=3;\ y^4 = 4y^3 = 4(3)(2)y' = 24(y')_0$
$n=4;\ y^5 = 5y4 = 5(4)(3)(2)y' = 5!(y')_0$
$n=5;\ y^6 = 6y^5 = 6(5)(4)(3)(2)y' = 6!(y')_0$
$n=6;\ y^7 = 7y^6 = 7(6)(5)(4)(3)(2)y' = 7!(y')_0$

Maclaurin Series

$$y = y_0 + x(y'_0) + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$$

$$= y^0 + x(y'_0) + \frac{x^2}{2!}(2y'_0) + \frac{x^3}{3!}(3!y'_0) + \frac{x^4}{4!}(4!y'_0) + \frac{x^5}{5!}(5!y'_0) + \frac{x^6}{6!}(6!y'_0) + \frac{x^7}{7!}(7!y'_0)$$

$$= y^0 + x(y')_0 + x^2(y')_0 + x^3(y')_0 + x^4(y')_0 + x^5(y')_0 + x^6(y')_0 + x^7(y')_0$$

$$= y^0 + y'(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Recall

$y' = y^0$

$y_0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$y' = 0.0005$ and $y^0 = 0.0005$, when $x=5$

$y^5 = 0.0005(1+5+5^2+5^3+5^4+5^5+5^6+5^7)$
$= 0.0005(97656)$
$= 48.828$
$\simeq 49$

when $x=8$, $y_0 = 0.0005$

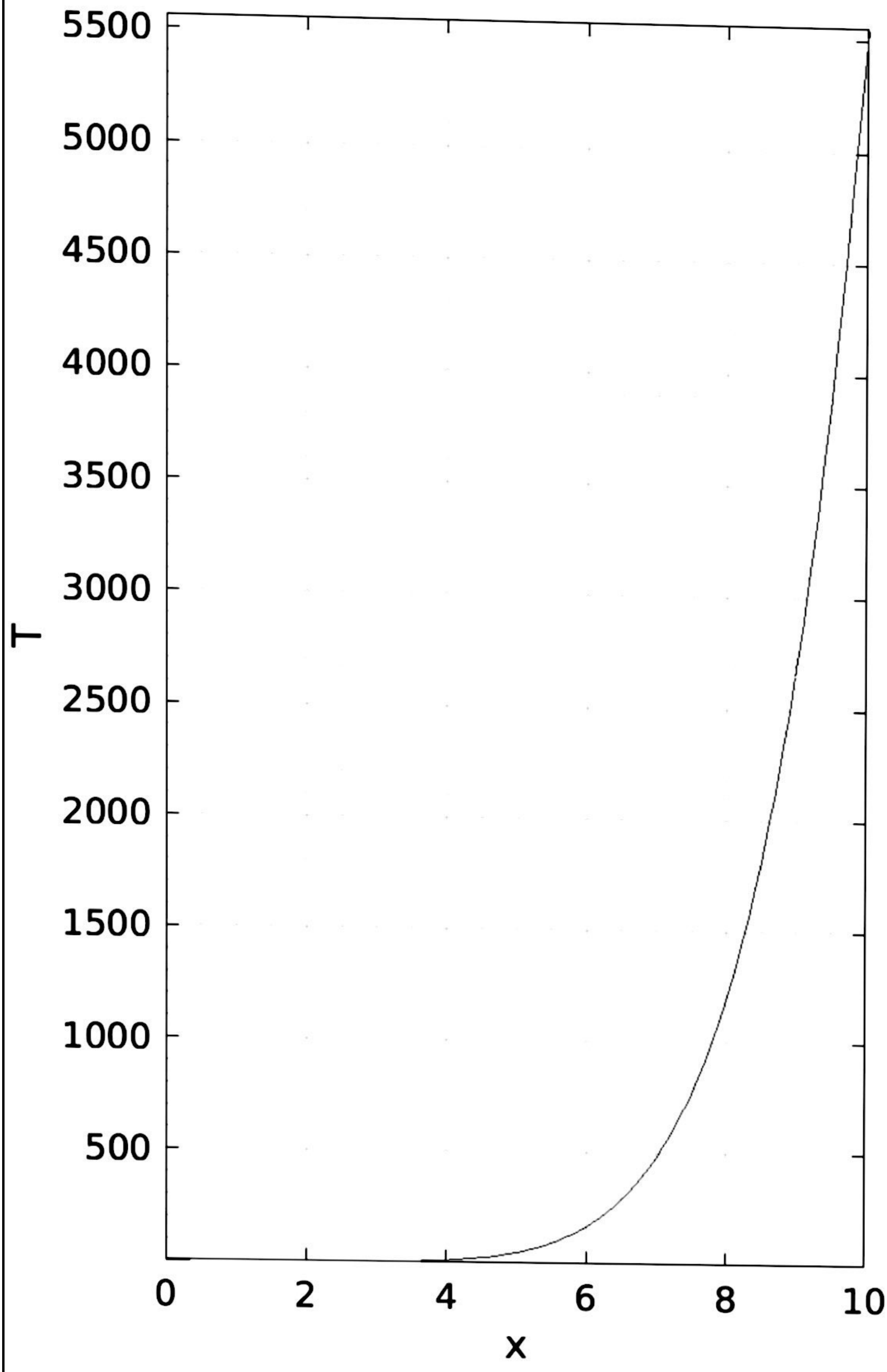
$y_8 = 0.0005(1+8+8^2+8^3+8^4+8^5+8^6+8^7)$
$= 0.0005(2396745)$
$= 1198.3725$
$\simeq 1198$

$x = 10$

$y_{10} = 0.0005(1+10+10^2+10^3+10^4+10^5+10^6+10^7)$
$= 0.0005(11111111)$
$= 5555.5555$
$\simeq 5556$

c Command Window

```
clc
clear all
close all
syms x y
x = 0:0.1:10
y = (0.0005)*(1+x+x^2+x^3+x^4+x^5+x^6+x^7)
yn = subs(y)
ynn = double(yn)
plot(x,ynn)
xlabel('X')
ylabel('T')
grid on
grid minor
axis tight
```



5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
T
0
2
4
6
8
10
x