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17/Eng02/022
Computer Engineering

1.) $x(x-1)y'' + (3x-1)y' + y = 0$

$(x^2-x)y'' + (3x-1)y' + y = 0$ Expanding bracket

we can then assume

$(x^2-x)y'' = W_1$ and $y = W_3$

$(3x-1)y' = W_2$

Using leibnitz theorem $= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$

W_1	W_2	W_3
let $u = y^n$ $v = x^2 - x$	let $u = y'$ $v = 3x-1$	let $u = y$ $v = 1$
$u^n = y^{n+2}$ $v' = 2x-1$	$u^n = y^{n+1}$ $v' = 3$	$u^n = y^n$ $v' = 0$
$u^{n-1} = y^{n+1}$ $v'' = 2$	$u^{n-1} = y^n$ $v'' = 0$	
$u^{n-2} = y^n$ $v''' = 2$		

$W_1 = (y^{n+2} \cdot (x^2-x)) + n(y^{n+1} \cdot (2x-1) + \frac{n(n-1)y^{n-2}}{2!}$

$W_2 = (y^{n+1} \cdot (3x-1) + n y^n \cdot 3 + 0$

$W_3 = y^n$

$\therefore W_1 + W_2 + W_3 = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + n(n-1)y^n + y^{n+1}(3x-1) + ny^n \cdot 3 + y^n = 0$

$0 = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + y^{n+1}(3x-1) + n(n-1)y^n + ny^n \cdot 3 + y^n = 0$

Assuming $x = 0$

$y^{n+1}(2(0)-n) + y^{n+1}(3(0)-1) + n(n-1)y^n + ny^n \cdot 3 + y = 0$

$= -ny^{n+1} - y^{n+1} + y^n(n^2-n) + 3ny^n + y^n = 0$

$= y^{n+1}(-n-1) + y^n(n^2-n+3n-1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2+2n-1) = 0$

$y^{(n+1)}(n+1) = y^n(n^2+2n-1)$

$y^{n+1} = y^n(n+1)$ $(n+1)$

$y^{n+1} = y^n(n+1)$ re-occulence relation

$(y^{n+1})_0 = y^n(n+1)_0$

$(y'')_0 = 0.0005$

$(y')_0 = 0.0005$

when $n = 0$ $(y^{(0+1)})_0 = (y^{(1)})_0 \; (0+1)$

$(y')_0 = (y^0)_0 = 1$

when $n=1$ $(y^{(1+1)})_0 = (y^1)_0 (1+1)$

$(y^2)_0 = (y^1)_0 2 = 2(y^1)_0$

when $n=2$ $(y^{2+1})_0 = (y^2)_0 (2+1)$

$(y^3)_0 = 3(y^2)_0 = 6(y^1)_0$

when $n=3$ $(y^{3+1})_0 = (y^3)_0 (3+1)$

$= (y^4)_0 = (y^3)_0 4 = 24 (y^1)_0$

when $n=4$ $(y^5)_0 = (y^4)_0 5 = 120 (y^1)_0$

when $n=5$ $(y^6)_0 = (y^5)_0 6 = 720 (y^1)_0$

when $n=6$ $(y^7)_0 = (y^6)_0 7 = 5040 (y^1)_0$

Using machevin series

$y = (y^0)_0 + x^1(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$

$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!} \cdot 2(y^1)_0 + \frac{x^3}{3!} 6(y^1)_0 + \frac{x^4}{4!} 24 (y^1)_0 + \frac{x^5}{5!} 120 (y^1)_0 + \frac{x^6}{6!} 720 (y^1)_0$

$+ \frac{x^7}{7!} 5040 (y^1)_0$

$y = (y^0)_0 (x+1) + (x^2+x^3+x^4+x^5+x^6+x^7)(y^1)_0$

$\therefore y = 0.0005(x+1) + (x^2+x^3+x^4+x^5+x^6+x^7) 0.0005$

(ii) when $x=5m$

$y = 0.0005(5+1) + (5^2+5^3+5^4+5^5+5^6+5^7) 0.0005$

$\therefore y = 48.828m$

when $x = 8m$

$y = 0.0005(8+1) + (8^2+8^3+8^4+8^5+8^6+8^7) 0.0005$

$\therefore y = 118.373m$

when $x=10$

$y = 0.0005(10+1) + (10^2+10^3+10^4+10^5+10^6+10^7) 0.0005$

$\therefore y = 5555.56m$

(iii) MATLAB file

Command window

clear

clc

close all

Close all

x = 0.001 : 10

y = (0.0005 * (x+1)) + ((x^2 + x^3 + x^4 + x^5 + x^6 + x^7) * (0.0005))

y_n = Subs (Y)

plot (x, y_n)

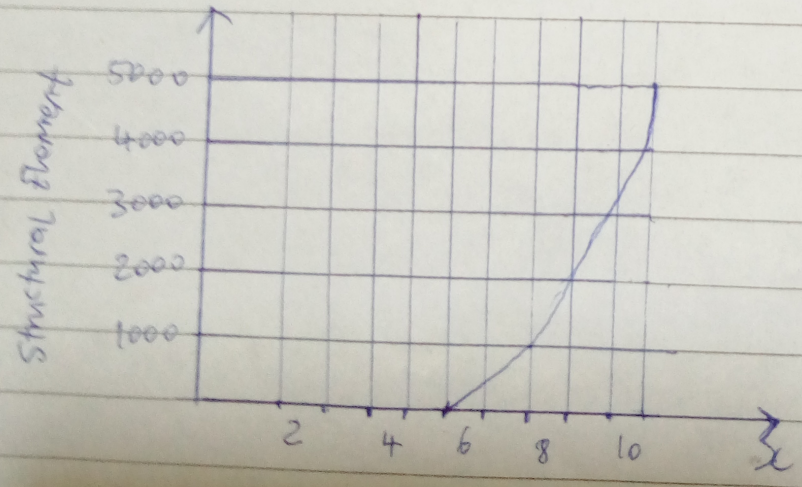
axis tight

grid on

grid minor

x label ('M')

Y label ('Deflection')

Sketch

Graph of Structural Element Against x