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Dept: MECHANICAL ENGINEERING

Mat no: HENG/06/1043

Course: Mathematical

Solution

$x(x-1)y'' + (3x-1)y' + y = 0$

Let

$W_1 = x(x-1)y''$

$W_2 = (3x-1)y'$

$W_3 = y$

For W_1

$u = y''$ $\quad v = x(x-1) = x^2 - x$

$u^n = y^{n+2}$ $\quad v' = 2x - 1$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^n$ $\quad v''' = 0$

For W_2

$u = y'$ $\quad v = 3x - 1$

$u^n = y^{n+1}$ $\quad v' = 3$

$u^{n-1} = y^n$ $\quad v'' = 0$

For W_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

Recall

$$y = uv + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v^2 + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v^3 + \cdots$$

$$y = y^{(n+2)}(x^2 - x) + n(y^{(n+1)})(2x-1) + \frac{n(n-1)}{2!}y^n \cdot 2 + \frac{n(n-1)(n-2)}{3!}(y^{(n-1)}) \cdot 0 + (y^{(n+1)})(3x-1)$$

$$+ ny^n \cdot 3 + \frac{n(n-1)}{2!}(y^{(n-1)}) \cdot 0 + y^n \cdot 1 + ny^{(n-1)} \cdot 0$$

$$= (x^2 - x)(y^{(n+2)}) + n(2x-1)(y^{(n+1)}) + n(n-1)(y^{(n)}) + (3x-1)(y^{(n+1)}) + n^3(y^n) + (y^n)$$

$$= (x^2 - x)(y^{(n+2)}) + (y^{(n+1)})(n(2x-1) + (3x-1)) + (y^{(n)})(n(n-1) + 3n + 1)$$

$$= (x^2 - x)(y^{(n+2)}) + (y^{(n+1)})(2nx - n + 3x - 1) + (y^n)(n^2 - n + 3n + 1)$$

$x=0$

$(y^{(n+2)})_0 - (n+1)(y^n)_0 (n^0+2n+1)=0$

$-(y^{(n+1)})_{(n+1)} + (y^{(n)})_0 (n^0+2n+1)=0$

$(y^{(n+1)})_{(n+1)} = (y^{(n)})_0 (n^0+2n+1)$

$(y^{(n+1)})_0 = (y^{(n)})_0 \frac{(n+1)(n+1)}{n+1}$

$(y^{(n+1)})_0 = (y^{(n)})_0 (n+1)$

Recall $(y^0)_0 = 0.0005$

$(y^{(0)})_0 = 0.0005$

$n=0$

$(y^{(0+1)})_0 = (y^0)(0+1)$

$(y^{(1)})_0 = 1(y^0)_0$

$n=1$

$y^{(1+1)})_0 = (y^1)(1+1)$

$y^{(2)})_0 = 2(y^1)_0$

$n=2$

$(y^{(2+1)})_0 = (y^2)(2+1)$

$(y^3)_0 = 3y^2 = 3x(y^2) = 6y^0$

$n=3$

$(y^{(3+1)})_0 = (y^3)(3+1)$

$(y^4)_0 = 4(y^3) = 4 \times 6(y^1) = 24(y^1)_0$

$n=4$

$(y^{(4+1)})_0 = (y^4)(4+1)$

$(y^5)_0 = 5(y^4) = 5 \times 24(y^1)_0 = 120(y^1)_0$

$n=5$

$(y^{(5+1)})_0 = (y^5)(5+1)$

$(y^6)_0 = 6(y^5) = 6 \times 120(y^1) = 720(y^1)_0$

$n=6$

$(y^{(6+1)})_0 = (y^6)(6+1)$

$y^7)_0 = 7(y^6) = 7 \times 720(y^1)_0 = 5040(y^1)_0$

Using Leibnitz Maclaurin theorem

$y = (y^{(0)})_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$

$y = (y^0)_0 + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{iv})_0 + \frac{x^5}{5!}(y^v)_0 + \frac{x^6}{6!}(y^{vi})_0 + \cdots$

$+ \cdots + \frac{x^n}{n!}(y^n)_0$

$y = y^0 + (1)(x) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(y')$

$y = 0.0005(1)(x) + 0.0005(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

When $x = 5$

$y = 0.0005(5) + 0.0005(5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$= 11.5254$ m

$x = 6$

$y = 0.0005(10) + 0.0005(6^2 + 6^3 + 6^4 + 6^5 + 6^6 + 6^7)$

$= 11.9835$ m

$x = 10$

$y = 0.0005(10) + 0.0005(10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$= 5555.5555$ m

Matlab file

Command window

clear

ac

close all

x = 0 : 0.01 : 10

y = (0.0005 * (1+x)) + (6x12 + x13 + x14 + x15 + x16 + x17) * 0.0005)

yn = fussy

Plot (x, yn)

• label ('m')

Y label ('Deflection')

axis on

grid on

grid minor