

$$(1) x(x-1)y'' + (3x-1)y' + y = 0$$

$$\text{Taking } x(x-1)y'' = w_1$$

$$(3x-1)y' = w_2$$

$$y = w_3$$

Considering w_1

$$U: y'' \quad V: x(x-1)$$

$$U': y^{(n+2)} \quad V': 2x-1$$

$$U^{n-1}: y^{(n+1)} \quad V'' = 2$$

$$U^{n-2}: y'' \quad V''' = 0$$

w_2

$$U: y' \quad V: (3x-1)$$

$$U': y^{(n+1)} \quad V' = 3$$

$$U'' = y'' \quad V'' = 0$$

w_3

$$U = y \quad V = 1$$

$$U' = y' \quad V' = 0$$

$$y^n: U^n V + n U^{(n-1)} V' + \frac{n(n-1)}{2!} U^{(n-2)} V'' + \frac{n(n-1)(n-2)}{3!} U^{(n-3)} V''' + \dots$$

$$w_1: y^{(n+2)} \cdot (x^2 - x) + n y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 = 0$$

$$w_2: y^{(n+1)} \cdot (3x-1) + n y^n \cdot 3 = 0$$

$$w_3: y'' \cdot 1 = 0$$

$w_1 + w_2 + w_3$

$$y^{(n+2)} \cdot (x^2 - x) + n y^{(n+1)} \cdot (3x-1)$$

$$(x^2-x)y^{(n+2)} + (2x-n)y^{(n+1)} + (3x-n)y^{(n)} + 3ay^{(n)} + y^{(n-1)}$$

$$(x^2-x)y^{(n+2)} + (2x-n)y^{(n+1)} + (3x-n)y^{(n)} + (n^2-2)y^{(n)} + 3ay^{(n)} + y^{(n-1)}$$

$$(x^2-x)y^{(n+2)} + (2x+n+3x-n-1)y^{(n+1)} + (n^2+n+3n+1)y^{(n)} = 0$$

$$(x^2-x)y^{(n+2)} + (2x+n+3x-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$(x-1)x y^{(n+2)} + (2x+n+3x-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

When $x=0$

$$(0-1)0 y^{(n+2)} + (2(0)+n+3(0)-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$(0-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$= (n+1) y^{(n+1)}$$

$$= (n+1) (y^{(n)})_0 + (n^2+2n+1) (y^{(n)})_0 = 0$$

$$(y^{(n+1)})_0 = -\frac{(n^2+2n+1) (y^{(n)})_0}{n+1}$$

$$(y^{(n+2)})_0 = (n+1)(n+1) (y^{(n)})_0$$

$$(y^{(n+1)})_0 = (n+1) (y^{(n)})_0$$

When $n=0$

$$(y^{(0)})_0 = (0+1) (y^{(0)})_0$$

$$(y^{(1)})_0 = 1 (y^{(0)})_0$$

When $n=1$

Ex

$$(y^{(2)})_0 = (1+1) (y^{(1)})_0$$

$$(y^{(2)})_0 = 2 (y^{(1)})_0$$

When $n=2$

$$[y^{(2)}]_0 = [2!] [y^{(2)}]_0$$

$$[y^{(3)}]_0 = 3 [y^{(2)}]_0 = 3(2!) [y^{(2)}]_0$$
$$[y^{(4)}]_0 = 6 [y^{(2)}]_0$$

When $n=3$

$$[y^{(4)}]_0 = (3!) [y^{(3)}]_0$$

$$[y^{(5)}]_0 = [y^{(4)}]_0$$

$$[y^{(6)}]_0 = 4 [y^{(5)}]_0 = 4[6] [y^{(5)}]_0$$

$$[y^{(7)}]_0 = 24 [y^{(5)}]_0$$

When $n=4$

$$[y^{(5)}]_0 = (4!) [y^{(4)}]_0$$

$$[y^{(6)}]_0 = 5 [y^{(5)}]_0 = 5(24) [y^{(4)}]_0$$

$$[y^{(7)}]_0 = 120 [y^{(4)}]_0$$

When $n=5$

$$[y^{(6)}]_0 = [5!] [y^{(5)}]_0$$

$$[y^{(7)}]_0 = 6 [y^{(6)}]_0 = 6(120) [y^{(4)}]_0$$

$$[y^{(8)}]_0 = 720 [y^{(4)}]_0$$

When $n=6$

$$[y^{(7)}]_0 = (6!) [y^{(6)}]_0$$

$$= 7 [y^{(7)}]_0 = 7(720) [y^{(4)}]_0$$

$$[y^{(8)}]_0 = 5040 [y^{(4)}]_0$$

$$\textcircled{a} y = [y^{(0)}]_0 + x [y^{(1)}]_0 + \frac{x^2}{2!} [y^{(2)}]_0 + \frac{x^3}{3!} [y^{(3)}]_0 + \frac{x^4}{4!} [y^{(4)}]_0$$
$$+ \frac{x^5}{5!} [y^{(5)}]_0 + \frac{x^6}{6!} [y^{(6)}]_0 + \frac{x^7}{7!} [y^{(7)}]_0$$

$$y = (y^{(0)})_0 + x [y^{(1)}]_0 + \frac{x^2}{2!} \cdot 2 [y^{(2)}]_0 + \frac{x^3}{3!} \cdot 6 [y^{(3)}]_0 +$$

$$\frac{x^4}{4!} \cdot 24 [y^{(4)}]_0 + \frac{x^5}{5!} \cdot 120 [y^{(5)}]_0 + \frac{x^6}{6!} \cdot 720 [y^{(6)}]_0 + \dots$$

$$\frac{x^7}{7!} \cdot 5040 [y^{(7)}]_0$$

$$\textcircled{a} y = (1+x) [y^{(0)}]_0 + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) [y^{(1)}]_0$$

$$y(0) = 0.0005 \text{ m}, \quad y'(0) = 0.0005$$

$$y = (1+x)(0.0005 \text{ m}) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(0.0005)$$

$$y = (1+x)(0.0005) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(0.0005)$$

When $x = 5 \text{ m}$, 8 m and 10 m

$$y = (1+5)(0.0005 \text{ m}) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)(0.0005)$$

$$y = 3 \times 10^{-3} \text{ m} + 97650 \text{ m} (0.0005)$$

$$y = 3 \times 10^{-3} \text{ m} + 48.825 \text{ m}$$

$$y = 48.828 \text{ m}$$

When $x = 8 \text{ m}$

$$y = (1+8)(0.0005) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)(0.0005)$$

$$y = 4.5 \times 10^{-3} + 2396736 (0.0005)$$

$$y = 4.5 \times 10^{-3} + 1198$$

$$y = 1198 \text{ m}$$

When $x = 10 \text{ m}$

$$y = (1+10)(0.0005) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)(0.0005)$$

$$y = 55 \times 10^{-3} + 1111100 (0.0005)$$

$$y = 555.5555 \text{ m}$$

$$y = 5556 \text{ m}$$

Matlab.

① Command Window

② `clear`

③ `clc`

4 `close all`

5 `syms x`

6 `x = ((11x) * (0.0005) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) *`
7 `(0.0005))`

8 `t = 0:0.01:10`

9 `xt = subs(x, t)`

10 `xtn = double(xt)`

11 `plot(t, xtn)`

12 `xlabel('t')`

13 `ylabel('x')`

14 `grid on`

15 `grid minor`

16 `axis tight`