

1.) $x(x-1)y'' + (3x-1)y' + y = 0$
 $(x^2-x)y'' + (3x-1)y' + y = 0$ Expanding bracket
 we can then assume

$(x^2-x)y'' = w_1$ and $y = w_3$

$(3x-1)y' = w_2$

using Leibnitz theorem $= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \dots$

let $u = y^n$ $v = x^2 - x$

$u^n = y^{n+2}$ $v' = 2x - 1$

$u^{n-1} = y^{n+1}$ $v'' = 2$

$u^{n-2} = y^n$ $v''' = 0$

let $u = y'$ $v = 3x - 1$

$u^n = y^{n+1}$ $v' = 3$

$u^{n-1} = y^n$ $v'' = 0$

let $u = y$ $v = 1$

$u^n = y^n$ $v' = 0$

$w_1 = (y^{n+2} - (x^2-x)) + n(y^{n+1} \cdot (2x-1)) + \frac{n(n-1)y^{n-2}}{2!}$

$w_2 = (y^{n+1} \cdot (3x-1)) + n y^n \cdot 3 + 0$

$w_3 = y^n$

$\therefore w_1 + w_2 + w_3 = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + n(n-1)y^n + y^{n+1}(3x-1) + ny^n - 3 + y^n = 0$

$\therefore y^{n+2}(x^2-x) + y^{n+1}(2x-n) + y^{n+1}(3x-1) + n(n-1)y^n + ny^n - 3 + y^n = 0$

assuming $x=0$

$y^{n+1}(2(0)-n) + y^{n+1}(3(0)-1) + n(n-1)y^n + ny^n - 3 + y^n = 0$

$= -ny^{n+1} - y^{n+1} + y^n(n^2-n) + 3ny^n + y^n = 0$

$= y^{n+1}(-n-1) + y^n(n^2-n+3n-1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2+2n-1) = 0$

$y^{(n+1)}(n+1) = y^n(n^2+2n-1)$

$y^{(n+1)}_{(n+1)} = y^n(n+1)(n+1)$

$y^{(n+1)} = y^n(n+1)$ re-occurrence relation

$(y^{(n+1)})_0 = y^n(n+1)$

$(y^{(n)})_0 = 0.0005$

$(y')_0 = 0.0005$

~~for~~ when $n=0$ $(y^{(6+1)})_0 = (y^{(6)})_0 (0+1)$

$(y')_0 = (y^0)_0 = 1$

when $n=1$ $(y^{(5+1)})_0 = (y^1)_0 = (y)_0$

$(y^2)_0 = (y^2)_0 = 2(y^1)_0$

when $n=2$ $(y^{(2+1)})_0 = (y^2)_0 = 2(y^1)_0$

$(y^3)_0 = 3(y^2)_0 = 6(y^1)_0$

when $n=3$ $(y^{(3+1)})_0 = (y^3)_0 = 3!(y^1)_0$

$(y^4)_0 = 4!(y^1)_0 = 24(y^1)_0$

when $n=4$ $(y^5)_0 = 5!(y^1)_0 = 120(y^1)_0$

when $n=5$ $(y^6)_0 = 6!(y^1)_0 = 720(y^1)_0$

when $n=6$ $(y^7)_0 = 7!(y^1)_0 = 5040(y^1)_0$

Using Maclaurin Series

$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0 + \dots$

$y = (y^1)_0 + x \left[(y^1)_0 + \frac{x}{2!} \cdot 2!(y^1)_0 + \frac{x^2}{3!} \cdot 6(y^1)_0 + \frac{x^3}{4!} \cdot 24(y^1)_0 + \frac{x^4}{5!} \cdot 120(y^1)_0 + \frac{x^5}{6!} \cdot 720(y^1)_0 + \frac{x^6}{7!} \cdot 5040(y^1)_0 \right]$

$y = (y^0)_0 (x+1) + (x^2+x^3+x^4+x^5+x^6+x^7) (y^1)_0$

$y = 0.0005(x+1) + (x^2+x^3+x^4+x^5+x^6+x^7) \cdot 0.0005$

(ii)

when $x=5m$

$y = 0.0005(5+1) + (5^2+5^3+5^4+5^5+5^6+5^7) \cdot 0.0005$

$y = 48.525m$

when $x=5m$

$y = 0.0005(4+1) + (4^2+4^3+4^4+4^5+4^6+4^7) \cdot 0.0005$

$y = 1184.733m$

when $x=10$

$y = 0.0005(10+1) + (10^2+10^3+10^4+10^5+10^6+10^7) \cdot 0.0005$

$y = 5555.56m$

iii) MH7 Lab m file

Common window

file

close all

Close all

$$x = 0.001:10$$

$$y = (0.0005 * (x+1)) + ((x^2 + x^3 + x^4 + x^5 + x^6 + x^7) * (0.0005))$$

$$y_n = \text{subs}(Y)$$

plot(x, y_n)

axis tight

grid on

grid minor

x label ('m')

y label ('Deflection')

Sketch

Graph of Structural Element Against x

