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~~Assignment III~~

Solution

1) $x(x-1)y'' + (3x-1)y' + y = 0$

for the nth derivative,

$x(n-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$

$x(x-1)y^{n+2} + y^{n+1}(2xn - n + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$

at $x = 0$,

$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 + 2n + 1) = 0$

$-(n+1)(y^{n+1})_0 = -(n^2+2n+1)(y^n)_0$

$(n+1)(y^{n+1})_0 = (n^2+2n+1)(y^n)_0$

at $n = 0$,

$(y^{0+1})_0 = (0+1)(y^0)_0$

$(y^1)_0 = (y)_0$

at $n = 1$

$(y^{1+1})_0 = (1+1)(y^1)_0$

$(y^2)_0 = 2(y^1)_0$

at $n = 2$,

$(y^{2+1})_0 = (2+1)(y^2)_0$

$(y^3)_0 = 3(y^2)_0$

$(y^3)_0 = 6(y^1)_0$

at $n = 3$,

$(y^4)_0 = (3+1)(y^3)_0$

$(y^4)_0 = 4(y^3)_0 = 4 \times 6(y^1)_0 = 24(y^1)_0$

at $n = 4$,

$(y^5)_0 = (4+1)(y^4)_0$

$= 5(y^4)_0$

$= 5 \times 24(y^1)_0$

$(y^5)_0 = 120(y^1)_0$

at $n=5$

$$(y^6)_0 = (5+1)(y^5)_0$$
$$= 6(y^5)_0$$
$$= 6 \times 120 (y')_0$$
$$(y^6)_0 = 720 (y')_0$$

at $n=6$,

$$(y^7)_0 = (6+1)(y^6)_0$$
$$= 7(y^6)_0$$
$$= 7 \times 720 (y')_0$$
$$(y^7)_0 = 5040 (y')_0$$

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \cdots$$

$$\therefore y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(2(y^2)_0) + \frac{x^3}{3!}(6(y')_0) + \frac{x^4}{4!}(24(y')_0)$$
$$+ \frac{x^5}{5!}(120(y')_0) + \frac{x^6}{6!}(720(y')_0) + \frac{x^7}{7!}(5040(y')_0) + \cdots$$

$$y = (y)_0 + x(y')_0 + x^2(y')_0 + x^3(y')_0 + x^4(y')_0 + x^5(y')_0 + x^6(y')_0 + x^7(y')_0 + \cdots$$

$$y = (y)_0 + (y')_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots]$$

but $(y)_0 = 0.0005$ m and $(y')_0 = 0.0005$,

$$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

b) When $x = 5$m,

$$y = 0.0005 + 0.0005 [5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$
$$y = 0.0005 + 0.0005 [97655]$$
$$\therefore y = 48.828 \text{ m}$$

When $x = 10$m,

$$y = 0.0005 + 0.0005 [10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000]$$

y = 0.00054 + 0.0005 [1 1 1 1 1 1 0]

y = 5555.5555 m

c) Command window

```
clear
dc
Syms x
Syms y
x = (0:10);
y = 0.0005 + 0.0005* (x+(x.^2)+(x.^3) + (x.^4) +(x.^5)
    (x.^6) + (x.^7))';
plot (x,y)
grid on
grid minor
xlabel ('x')
ylabel ('Stretched Elongation')
```