Let $y = W_3$

$W_3^{(n)} = y^{(n)}$

∴ nth derivative;

$y^{(n+2)} = (x^2-x) + ny^{(n+1)} \cdot (2x-1) + n(n-1)y^{(n)} + y^{(n+1)}$

$\cdot (3x-1) + ny^{(n)} \cdot 3 + y^{(n)} = 0$

Assuming $x=0$

$n(n-1)y^{(n)} + 3ny^{(n)} + y^{(n)} - ny^{(n+1)} - y^{(n+1)} = 0$

$(n^2+2n+1)y^{(n)} - (n+1)y^{(n+1)} = 0$

$y^{(n+1)} = \frac{(n^2+2n+1)y^{(n)}}{(n+1)} = \frac{(n+1)(n+1)y^{(n)}}{(n+1)}$

Recurrence relation = $y^{(n+1)} = (n+1)y^{(n)}$

When $n=0$; $(y^{(1)})_0 = (y^{(0)})_0$

$n=1$; $(y^{(2)})_0 = 2(y^{(1)})_0 = 2(y^{(0)})_0$

$n=2$; $(y^{(3)})_0 = 3(y^{(2)})_0 = 3\times2(y^{(0)})_0 = 6(y^{(0)})_0$

$n=3$; $(y^{(4)})_0 = 4(y^{(3)})_0 = 4\times6(y^{(0)})_0 = 24(y^{(0)})_0$

$n=4$; $(y^{(5)})_0 = 5(y^{(4)})_0 = 5\times24(y^{(0)})_0 = 120(y^{(0)})_0$

$n=5$; $(y^{(6)})_0 = 6(y^{(5)})_0 = 6\times120(y^{(0)})_0 = 720(y^{(0)})_0$

$n=6$; $(y^{(7)})_0 = 7(y^{(6)})_0 = 7\times720(y^{(0)})_0 = 5040(y^{(0)})_0$

Maclaurin Series;

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \dots$

$\therefore y = (y^{(0)})_0 + x(y^{(0)})_0 + \frac{x^2}{2!}(2(y^{(0)})_0) + \frac{x^3}{3!}(6y^{(0)})_0 + \dots$

$+ \frac{x^4}{4!}(24(y^{(0)})_0) + \frac{x^5}{5!}(120(y^{(0)})_0) + \frac{x^6}{6!}(720(y^{(0)})_0) +$

$\frac{x^7}{7!}(5040(y^{(0)})_0) + \dots$

$y = (y^{(0)})_0 + x(y^{(0)})_0 + x^2(y^{(0)})_0 + x^3(y^{(0)})_0 + x^4(y^{(0)})_0 + \dots$

c) MATLAB mfile

- Command window
- Clear
- Clc
- Close all
- x=0:0.01:10
- y=0.0005*(1+x+x.^2+x.^3+x.^4+x.^5
  +x.^6+x.^7).
- yn = subs(y)
- Plot(x, yn)
- xlabel('m')
- ylabel('Deflection')
- axes tight
- grid on
- grid minor

$x^5(y^{(0)})_0 + x^6(y^{(0)})_0 + x^7(y^{(0)})_0 + \dots$

a) Power Series: $y = (y^{(0)})_0(1+x+x^2+x^3+x^4+x^5+x^6+ x^7+\dots)$

Since $(y^{(0)})_0 = 0.0005m$ and $(y^{(0)})_0 = 0.0005$

Power Series: $y = 0.0005(1+x+x^2+x^3+x^4+x^5+x^6 +x^7+\dots)$.

b) When $x = 5m$

$y = 0.0005(1+x+x^2+x^3+x^4+x^5+x^6+x^7+\dots)$

$y = 0.0005(1+5+5^2+5^3+5^4+5^5+5^6+5^7)$

$y = 48.828m$

when $x = 8m$

$y = 0.0005(1+x+x^2+x^3+x^4+x^5+x^6+x^7+\dots)$

$y = 0.0005(1+8+8^2+8^3+8^4+8^5+8^6+8^7)$

$y = 1198.3725m$

When $x = 10m$

$y = 0.0005(1+x+x^2+x^3+x^4+x^5+x^6+x^7+\dots)$

$y = 0.0005(1+10+10^2+10^3+10^4+10^5+10^6+10^7)$

$y = 5555.5555m$

$x(x-1)y'' + (3x-1)y' + y = 0$

Applying leibnitz - maclaurin method

$(x^2 - x)y'' + (3x-1)y' + y = 0$

$(x^2 - x)y^{(2)} + (3x-1)y^{(1)} + y = 0$

Find the nth derivative using leibnitz theorem.

Let $(x^2 - x)y^{(2)} = W_1$

$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \dots$

$u = y^{(2)}$ $\quad v = x^2 - x$

$u^{(n)} = y^{(n+2)}$ $\quad v' = 2x-1$

$u^{(n-1)} = y^{(n+1)}$ $\quad v'' = 2$

$u^{(n-2)} = y^{(n)}$ $\quad v''' = 0$

$W_1^{(n)} = y^{(n+2)}(x^2 - x) + ny^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2!}y^{(n)} \cdot \cancel{2}$

Let $(3x-1)y^{(1)} = W_2$

$u = y^{(1)}$ $\quad v = 3x-1$

$u^{(n)} = y^{(n+1)}$ $\quad v' = 3$

$u^{(n-1)} = y^{(n)}$ $\quad v'' = 0$

$W_2^{(n)} = y^{(n+1)} \cdot (3x-1) + ny^{(n)} \cdot 3$