AbdulAzeez Zubaidat
17/engineering/?
Civil Engineering

a) $x(x-1)y'' + (3x-1)y' + y = 0$

Expanding the brackets

$(x^2-x)y'' + (3x-1)y' + y = 0$

$w_1 = (x^2-x)y''$: $w_2 = (3x-1)y'$: $w_3 = y$

Using Leibnitz for w_1, w_2 and w_3

${}^nC_0 v^{(n)} v^{(0)} + {}^nC_1 v^{(n-1)} v^{(1)} + {}^nC_2 v^{(n-2)} v^{(2)} + \dots$

w_1

$u = y^{(2)}$ $\qquad v = x^2 - x$

$u^n = y^{(2+n)}$ $\qquad v^{(1)} = 2x - 1$

$v^{(n+1)} = y^{(1+n)}$ $\qquad v^{(2)} = 2$

$v^{(n-2)} = y^{(n)}$ $\qquad v^{(3)} = 0$

$w_1 = y^{(n+2)}$

$w_1 = y^{(n+2)} \cdot (x^2-x) + n \cdot y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2} y^{(n)} \cdot 2$

w_2

$u = y^{(1)}$ $\qquad v = 3x - 1$

$u^n = y^{(n+1)}$ $\qquad v^{(1)} = 3$

$u^{(n-1)} = y^{(n)}$ $\qquad v^{(2)} = 0$

$w_2 = y^{(n+1)} \cdot (3x-1) + 3ny^{(n)}$

w_3

$u = y$ $\qquad v = 1$

$u^{(n)} = y^{(n)}$ $\qquad v^{(1)} = 0$

$w_3 = y^n$

$w_1 + w_2 + w_3$

$y^{(n+2)} \cdot (x^2-x) + (2nx-n)y^{(n+1)} + y^{(n)}(n^2-n) + y^{(n+1)}(3x-1)$
$+ 3ny^{(n)} + y^{(n)} = 0$

Assume $x = 0$

$(-n)\,y^{(n+1)} + y^{(n)}(n^2-n) + 3n\,y^{(n)} + y^{(n)} = 0$

Considering like term

$y^{(n+1)}(-n-1) + y^{(n)}(n^2-n+3n+1) = 0$

$y^{(n+1)}(-n-1) + y^{(n)}(n^2+2n+1) = 0$

$-y^{(n+1)}(n+1) + y^{(n)}(n^2+2n+1) = 0$

$y^{(n+1)}(n+1) = y^{(n)}(n^2+2n+1)$

$y^{(n+1)}\cancel{(n+1)} = y^{(n)}\cancel{(n+1)}(n+1)$

$(y^{(n+1)})_0 = (y^{(n)})_0\,(n+1)$ (recurrent relations)

Assume $n=0$

$(y^{(1)})_0 = (y^{(0)})_0\,(1)$

$0.005 = 0.005\,(1)$

$\Rightarrow n=1$

$(y^{(2)})_0 = 2(y^{(1)})_0$

$\Rightarrow n=2$

$(y^{(3)})_0 = 3(y^{(2)})_0 \Rightarrow 3(2(y^{(1)}))_0 = 6(y^{(1)})_0$

$\Rightarrow n=3$

$(y^{(4)})_0 = 4(y^{(3)})_0 = 4(3(2(y^{(1)}))) = 24(y^{(1)})_0$

$\Rightarrow n=4$

$(y^{(5)})_0 = 5(y^{(4)})_0 = 5(4(3(2(y^{(1)})_0))) = 120(y^{(1)})_0$

$\Rightarrow n=5$

$(y^{(6)})_0 = 6(y^{(5)})_0 \Rightarrow 6(5(4(3(2(y^{(1)})_0)))) = 720(y^{(1)})$

$\Rightarrow n=6$

$(y^{(7)})_0 = 7(y^{(6)})_0 \Rightarrow 7(6(5(4(3(2(y^{(1)})_0))))) = 5040(y$

maclaurins Series

$y = (y^{(0)})_0 + (y^{(1)})_0 x + \frac{(y^{(2)})_0}{2!}x^2 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 +$

$\frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0$

$\Rightarrow y = 0.0005 + 0.0005x + \frac{2x^2}{2}(y^{(1)})_0 + \frac{6x^3}{3!}(y^{(1)})_0 + \frac{24x^4}{4!}$

$(y^{(1)})_0 + \frac{120x^5}{5!}(y^{(1)})_0 + \frac{720x^6}{6!}(y^{(1)})_0 + \frac{5040x^7}{7!}(y^{(1)})_0$

$y = (y^{(0)})_0 + x(y^{(0)})_0 + x^2(y^{(0)})_0 + x^3(y^{(0)})_0 + x^4(y^{(0)})_0 + \ldots$

$y = (y^{(0)})_0 + x(y^{(0)})_0 + x^2(y^{(0)})_0 + \ldots$

$x^6(y^{(0)})_0 + x^7(y^{(0)})_0 + \ldots$

(a) Power series $y = (y^{(0)})_0 (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \ldots)$ and $(y^{(0)})_0 = 0.0005$

Since $(y^{(0)})_0 = 0.0005\,m$

Power Series : $y = 0.0005\,(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \ldots)$

(b) when $x = 5\,m$

$y = 0.0005\,(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$y = 0.0005\,(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$y = 48.828\,m$

when $x = 8\,m$.

$y = 0.0005\,(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \ldots)$

$y = 0.0005\,(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$y = 1198.3725\,m$.

when $x = 10\,m$

$y = 0.0005\,(1 + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \ldots)$

$y = 0.0005\,(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7 + \ldots)$

$y = 5555.5555\,m$.

(c) MATLAB FILE

Command window

```
clear
clc
close all
x = 0 : 0.01 : 10
y = 0.0005 * (1 + x + x^2 + x.^3 + x.^4 + x.^5 + x.^6 + x.^
yn = subs (y)
Plot (x, yn)
xlabel ('m')
```

grid on

grid minor