

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Part a

$$x(x-1)y'' , u = y'' , u^n = y^{n+2}$$

$$v = 3x-1 , dv = 3dx , v^n = 3^n$$

$$y' = u'x + nu^{n-1}v' + n(x-1)u^{n-2}y''$$

$$= x(x-1)y^{n+2} + ny^{n+1} \cdot (3x-1) + n(x-1)y^{n+2}$$

$$= x(x-1)y^{n+2} + ny^{n+1}(3x-1) + (n^2 - n)x^2y^n$$

Part b

$$(3x-1)y' , u = y' , u^n = y^{n+1}$$

$$v = 3x-1 , v' = 3$$

$$y^n = y^n u + nu^{n-1}v'$$

$$= (3x-1)y^{n+1} + n3y^n$$

Part c

$$y' , u = y' , u^n = y^n$$

$$\therefore \text{Part A} + \text{Part B} + \text{Part C} = 0$$

$$x(x-1)y^{n+2} + ny^{n+1}(3x-1) + (n^2 - n)x^2y^n + (3x-1)y^{n+1} + 3ny^n + y'$$

$$x(x-1)y^{n+2} + (3x-1)(n+1)y^{n+1} + (n^2 + 2n + 1)x^2y^n = 0$$

$$\text{when } n=0$$

$$(3x-1)(n+1) + (3x-1)(n+1) + (n^2 + 2n + 1)x^2 = 0$$

$$- (n+1)(3x-1) = - [(n+1)(3x-1)] y^n = 0$$

$$\therefore y^{n+1} = (n+1)y^n$$

$$\text{when } n=0$$

$$y_0' = y_0$$

$$\text{when } n=1$$

$$y_0'' = 2y_0' , 2(y_0')$$

$$\text{when } n=2$$

$$y_3^3 = 5y_0^3, \quad 3(2y_0^3) = 6y_0^3$$

when $n = 3$

$$y_4^4 = 4y_0^4 + (6y_0^3) = 24y_0^4$$

when $n = 4$

$$y_5^5 = 5y_0^5 + 5(24y_0^4) = 120y_0^5$$

when $n = 5$

$$y_6^6 = 6y_0^6 + 6(120y_0^5) = 720y_0^6$$

when $n = 6$

$$y_7^7 = 7y_0^7 + 7(720y_0^6) = 5040y_0^7$$

when $n = 7$

From Maclaurin Series

$$y = \frac{x^0 y_0}{0!} + \frac{x^1 y_1}{1!} + \frac{x^2 y_2}{2!} + \frac{x^3 y_3}{3!} + \frac{x^4 y_4}{4!} + \frac{x^5 y_5}{5!} + \frac{x^6 y_6}{6!} + \frac{x^7 y_7}{7!}$$

$$y = y_0 + 2xy_0 + \frac{x^2 24y_0}{2!} + \frac{x^3 64y_0}{3!} + \frac{x^4 244y_0}{4!} + \frac{x^5 1204y_0}{5!} + \frac{x^6 7204y_0}{6!} + \frac{x^7 50404y_0}{7!}$$

$$y = y_0 + 2xy_0 + x^2 y_0 + x^3 y_0 + x^4 y_0 + x^5 y_0 + x^6 y_0 + x^7 y_0 + \dots$$

$$y = y_0 [1 + 2x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots]$$

where $y_0 = 0.0005$ $\frac{1}{2} y_0 = 0.00025$

where $y_0 = y_0'$

b)

when $x = 5$

$$y = 0.0005 [1 + 5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$

$$y = 0.0005 [97656]$$

$$y = 48.828$$

when $x = 8$

$$y = 0.0005 [1 + 8 + 64 + 512 + 4096 + 32768 + 262144 + 2,097,152]$$

$$y = 0.0005 [2,396,745]$$

$$y = 1198.3725$$

When $x = 10$

$$y = 0.0005 [1 + 10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000]$$

$$y = [11, 11, 11] \cdot 0.0005$$

$$y = 5,555.555$$

c) Command window

clear

clc

Syms x

Syms y

$$x = (0:10);$$

$$y = 0.0005 (1 + x + (x.^2)) + (x.^3) + (x.^4) + (x.^5) + (x.^6) + (x.^7);$$

Plot (x,y)

grid on

grid minor

x label ('x')

y label ('structural deformation')