

GUMOR-NUKI WARE-EBI HARPT

17/ENG06/038

MECHANICAL ENGINEERING

ENG 381

### ASSIGNMENT 3

$$x(x-1)y'' + (3x-1)y' + y = 0$$

For the  $n$ th derivative,

$$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$x(x-1)y^{n+2} + y^{n+1}(2x-1+n) + y^n(n^2-n+3n+1) = 0$$

at  $x=0$ ,

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2+2n+1) = 0$$

$$-(n+1)(y^{n+1})_0 = -(n^2+2n+1)(y^n)_0$$

$$(n+1)(y^{n+1})_0 = (n^2+2n+1)(y^n)_0$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

at  $n=0$

$$(y^{0+1})_0 = (0+1)(y^0)_0$$

$$(y^1)_0 = (y^0)_0$$

at  $n=1$

$$(y^{1+1})_0 = (1+1)(y^1)_0$$

$$(y^2)_0 = 2(y^1)_0$$

at  $n=2$

$$(y^{2+1})_0 = (2+1)(y^2)_0$$

$$(y^3)_0 = 3(y^2)_0$$

$$(y^3)_0 = 6(y^1)_0$$

at  $n=3$

$$(y^{3+1})_0 = (3+1)(y^3)_0$$

$$(y^4)_0 = 4(y^3)_0 = 4 \times 6(y^1)_0 = 24(y^1)_0$$

at  $n=4$

$$(y^4)_0 = (4+1)(y^4)_0$$

$$= 5(y^4)_0$$

$$= 5 \times 24 (y')_0$$

$$(y^5)_0 = 120 (y')_0$$

at  $n = 5$

$$(y^6)_0 = (5+1) (y^5)_0$$

$$= 6 (y^5)_0$$

$$= 6 \times 120 (y')_0$$

$$(y^6)_0 = 720 (y')_0$$

at  $n = 6$ ,

$$(y^7)_0 = (6+1) (y^6)_0$$

$$= 7 (y^6)_0$$

$$= 7 \times 720 (y')_0$$

$$(y^7)_0 = 5040 (y')_0$$

From maclaurin series,

$$y = (y)_0 + x (y')_0 + \frac{x^2}{2!} (y'')_0 + \frac{x^3}{3!} (y''')_0 + \frac{x^4}{4!} (y^{(4)})_0 + \dots$$

$$\therefore y = (y)_0 + x (y')_0 + \frac{x^2}{2!} (2 (y')_0) + \frac{x^3}{3!} (6 (y')_0) + \frac{x^4}{4!} (24 (y')_0)$$

$$+ \frac{x^5}{5!} (120 (y')_0) + \frac{x^6}{6!} (720 (y')_0) + \frac{x^7}{7!} (5040 (y')_0) + \dots$$

$$y = (y)_0 + x (y')_0 + x^2 (y')_0 + x^3 (y')_0 + x^4 (y')_0 + x^5 (y')_0 + x^6 (y')_0 + x^7 (y')_0 + \dots$$

$$y = (y)_0 + (y')_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots]$$

$$\text{but } (y)_0 = 0.0005 \text{ m and } (y')_0 = 0.0005$$

$$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

b) When  $x = 5 \text{ m}$ ,

$$y = 0.0005 + 0.0005 [5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$

$$y = 0.0005 + 0.0005 [97655]$$

$$\therefore y = 48.828 \text{ m}$$

When  $x = 8m$ ,

$$y = 0.0005 + 0.0005 [8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152]$$

$$y = 0.0005 + 0.0005 [2396744]$$

$$\therefore y = 1198.3725m$$

When  $x = 10m$ ,

$$y = 0.0005 + 0.0005 [10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000]$$

$$y = 0.0005 + 0.0005 [11111110]$$

$$y = 5555.5555m$$

c. Command Window

```
clear
```

```
clc
```

```
syms x
```

```
syms y
```

```
x = (0:10);
```

```
y = 0.0005 + 0.0005 * (x + (x.^2) + (x.^3) + (x.^4) + (x.^5) + (x.^6) + (x.^7))
```

```
plot(x,y)
```

```
grid on
```

```
grid minor
```

```
xlabel('x')
```

```
ylabel('Structural Deformation')
```