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BIOMEDICAL ENGINEERING.
17/MHS01/089.
ENG 881 - ASSIGNMENT III

QUESTION.

$x(x-1)y'' + (3x-1)y' + y = 0$

$(x^2-x)y'' + (3x-1)y' + y = 0$

$A + B + C = 0$

LEIBNITZ - MACLAURIN'S METHOD.

$$y = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v'' + \frac{n(n-1)(n-2)}{3!}u^{n-3}v''' + \frac{n(n-1)(n-2)(n-3)}{4!}u^{n-4}v^4 + \dots$$

$A = (x^2-x)y''$

$u = y''$; $u^n = y^{2+n}$; $u^{n-1} = y^{1+n}$; $u^{n-2} = y^n$

$v = x^2 - x$ $v' = 2x-1$; $v'' = 2$; $v''' = 0$

$$A^n = y^{n+2}(x^2-x) + n(y^{1+n})(2x-1) + \frac{n(n-1)}{2!}y^n \cdot 2 + 0$$

$$A^n = y^{n+2}(x^2-x) + n(y^{1+n})(2x-1) + (n^2-n)y^n + 0$$

$B = (3x-1)y'$

$u = y'$; $u^n = y^{1+n}$, $u^{n-1} = y^n$

$v = 3x-1$ $v' = 3$ $v'' = 0$

$$B^n = y^{1+n}(3x-1) + 3ny^n + 0$$

$C = y$

$C^n = y^n$.

$A + B + C = 0$

$$y^{n+2}(x^2-x) + ny^{1+n}(2x-1) + (n^2-n)y^n + y^{1+n}(3x-1) + 3ny^n + y^n = 0$$

Collecting like terms.

$$0 = y^{n+2}(x^2-x) + y^{1+n}(2nx-n+3x-1) + y^n(n^2-n+3n+1)$$

at $x = 0$

$$= (y^{n+2})_0(0^2-0) + (y^{1+n})(2n(0)-n+3(0)-1) + y^n(n^2+0) + (y^n)_0(n^2+2n+1)$$

$$= (y^{1+n})(-n-1) + (y^n)_0(n^2+2n+1)$$

$$= -y^{1+n}(n+1) + (y^n)_0(n^2+2n+1)$$

$$(y^{1+n})(n+1) = (y^n)_0(n^2+2n+1)$$

$$(y^{1+n})_0 = \frac{(n^2+2n+1)y^n}{(n+1)}$$

n^2+2n+1

$n^2 = n, n$.

$$n^2+n+n+1$$

$$n(n+1)+1(n+1) = (n+1)(n+1)$$

$$\therefore y^{1+n} = \frac{(n+1)(n+1)y^n}{(n+1)}$$

$$\boxed{y^{1+n} = (n+1)(y^n)_0}$$ → Reoccuring Equation.

at $n = 0$

$$(y')_0 = (y^0)_0$$

when $n = 1$

$$(y'')_0 = 2(y')_0$$

when $n = 2$

$(y'''')_0 = 5(y''')_0 = 6(y'')_0$

when $n = 3$

$(y^4)_0 = 4(y''')_0 = 24(y')_0$

when $n = 4$

$(y^5) = 5(y^4)_0 = 120(y')_0$

when $n = 5$

$(y^6)_0 = 6(y^5)_0 = 720(y')_0$

when $n = 6$

$(y^7)_0 = 7(y^6)_0 = 5040(y')_0$.

a. Power Series.

$$y(y^0)_0 + x(y')_0 + \frac{x^2(y'')_0}{2!} + \frac{x^3(y''')_0}{3!} + \frac{x^4(y^4)_0}{4!} + \frac{x^5(y^5)_0}{5!} + \frac{x^6(y^6)}{6!} + \frac{x^7(y^7)_0}{7!} + \ldots$$

$$y = (y)_0 + x(y^0)_0 + \frac{x^2}{2!}(y')_0 \cdot 2 + \frac{x^3}{3!} \cdot 6(y')_0 + \frac{x^4}{4!} \cdot 24(y')_0 + \frac{x^5}{5!} \cdot 120(y')_0 + \frac{x^6}{6!} \cdot 720$$

$$+ \frac{x^7}{7!} \cdot 5040(y')_0$$

$$y = (y_0)(1+x) + (y')_0[x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

Given that $(y^0)_0 = 0.0005m$

$(y')_0 = 0.005$

$$y = 0.0005m(1+x) + 0.0005(x^0 + x^3 + x^4 + x^6 + x^7)$$

b. When $x = 5m, 8m$ and $10m$

$$\therefore y = 0.0005m(1+5) + 0.0005(5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$$

$$y = 0.0005m(6) + 0.0005(25 + 125 + 625 + 3,125 + 15,625 + 78,125)$$

$$y = 0.0005m(6) + 0.0005(97650m)$$

$$y = 0.0005m(6) + 48.825m$$

$Y = 48.805m \approx 49m$

when $x = 8m$

$Y = 0.0005m(78) + 0.0005(8^0 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$Y = 0.0005m(9) + 0.0005(64 + 512 + 4.096 + 32,768 + 262,144 + 2,097,152)$

$Y = 0.0005m(9) + 0.0005(2,396,736)$

$= 0.0045 + 1,198,368m$

when $x = 10m$

$Y = 0.0005(1+10) + 0.0005(10^0 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$Y = 0.0005(11) + 0.0005(100 + 1000 + 10000 + 100,000 + 1000000 + 10000000)$

$Y = 0.0005m(11) + 0.0005(11,111,100m)$

$= 0.0055m + 5555.55m$

$Y = 5,555.5555m \approx 5,556m \approx 556m$

C. Command Window

```
clear
clc
close all
Sym x
x = [(1+x)*(0.0005)] + [(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)*(0.0005)]

t = 0:0.01:10
xt = Subs(x,t)
xtn = double(xt).
plot(t,xtn)
xlabel('t')
ylabel('x')
Grid on
Grid minor
axis tight
```