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 COURSE: ENG 381

①  $x(x-1)y'' + (3x-1)y' + y = 0$

Taking  $x(x-1)y'' = w_1$   
 $(3x-1)y' = w_2$   
 $y = w_3$

Considering  $w_1$ ,

$u = y''$   
 $u^n = y^{(n+2)}$   
 $u^{n-1} = y^{(n+1)}$   
 $u^{n-2} = y^2$

$V = x(x-1)$   
 $V' = 2x-1$   
 $V'' = 2$   
 $V''' = 0$

$w_2$

$u = y'$   
 $u^n = y^{(n+1)}$   
 $u^{n-1} = y$

$V = (3x-1)$   
 $V' = 3$   
 $V'' = 0$

$w_3$

$u = y$   
 $u^n = y^n$

$V = 1$   
 $V = 0$

$y^n = u^n V + \frac{n(n-1)}{2!} u^{(n-2)} V' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} V'' + \dots$

$w_1^n = y^{n+2} \cdot (x^2-x) + n y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2!} y^{n-2} \cdot 2 + 0$

$w_2^n = y^{(n+1)} \cdot (3x-1) + n y^n \cdot 3 + 0$

$w_3^n = y^n \cdot 1 + 0$

$w_1 + w_2 + w_3$

$y^{n+2} \cdot (x^2-x) + n y^{n+1} \cdot (2x-1) + \frac{n(n-1)}{2} y^{n-2} \cdot 2 + y^{(n+1)} \cdot (3x-1)$

$n y^n = 3 + y^n = 0$

$$(x^2 - x) y^{(n+2)} + (2x-1) n y^{(n+1)} + (3x-1) y^{n+1} + n(n-1) y^n + 3n y^n + y^n = 0$$

$$(x^2 - x) y^{(n+2)} + (2xn - n) y^{(n+1)} + (3x-1) y^{(n+1)} + (n^2 - 2) y^n + 3n y^n + y^n = 0$$

$$(x^2 - x) y^{(n+2)} + (2xn + 3x - n - 1) y^{(n+1)} + (n^2 - n + 3n + 1) y^n = 0$$

$$(x^2 - x) y^{(n+2)} + (2xn + 3x - n - 1) y^{n+1} + (n^2 + 2n + 1) y^n = 0$$

$$(x-1) x y^{n+2} + (2xn + 3x - n - 1) y^{n+1} + (n^2 + 2n + 1) y^n = 0$$

when  $x = 0$

$$(0-1) y^{n+2} + (2(0)n + 3(0) - n - 1) y^{n+1} + (n^2 + 2n + 1) y^n = 0$$

$$(-n-1) y^{n+1} + (n^2 + 2n + 1) y^n = 0$$

$$-(n+1) (y^{n+1})_0 + (n^2 + 2n + 1) (y^n)_0 = 0$$

$$(y^{n+1})_0 = \frac{(n^2 + 2n + 1) (y^n)_0}{n+1}$$

$$(y^{n+1})_0 = \frac{(n+1)(n+1)}{(n+1)} (y^n)_0$$

$$(y^{n+1})_0 = (n+1) (y^n)_0$$

when  $n = 0$

$$(y^{0+1})_0 = (0+1) (y^0)_0$$

$$(y^1)_0 = 1 (y^0)_0$$

when  $n = 1$

$$(y^{1+1})_0 = (1+1) (y^1)_0$$

$$(y^{(2)})_0 = 2 (y^{(1)})_0$$

When  $n=2$

$$[y^{(3)}]_0 = [2+1] [y^{(2)}]_0$$

$$[y^{(3)}]_0 = 3[y^{(3)}]_0 = 3(2) [y^{(2)}]_0$$
$$[y^{(3)}]_0 = 6 [y^{(2)}]_0$$

When  $n=3$

$$[y^{(4)}]_0 = 3+1 [y^{(3)}]_0$$

$$[y^{(4)}]_0 = 4 [y^{(3)}]_0 = 4 [6] [y^{(1)}]_0$$

$$[y^{(4)}]_0 = 24 [y^{(1)}]_0$$

When  $n=4$

$$[y^{(5)}]_0 = (4+1) [y^{(4)}]_0$$

$$[y^{(5)}]_0 = 5 [y^{(4)}]_0 = 5(24) [y^{(1)}]_0$$

$$[y^{(5)}]_0 = 120 [y^{(1)}]_0$$

When  $n=5$

$$[y^{(6)}]_0 = [5+1] [y^{(5)}]_0$$

$$[y^{(6)}]_0 = 6 [y^{(5)}]_0 = 6 [120] [y^{(1)}]_0$$

$$[y^{(6)}]_0 = 720 [y^{(1)}]_0$$

When  $n=6$

$$[y^{(7)}]_0 = (6+1) [y^{(6)}]_0$$

$$= 7 [y^{(6)}]_0 = 7 (720) [y^{(1)}]_0$$

$$[y^{(7)}]_0 = 5040 [y^{(1)}]_0$$

$$y = [y^{(1)}]_0 + x [y^{(1)}]_0 + \frac{x^2}{2!} [y^{(2)}]_0 + \frac{x^3}{3!} [y^{(3)}]_0 + \frac{x^4}{4!} [y^{(4)}]_0$$

$$+ \frac{x^5}{5!} [y^{(5)}]_0 + \frac{x^6}{6!} [y^{(6)}]_0 + \frac{x^7}{7!} [y^{(7)}]_0$$

$$y = (y^{(1)})_0 + x [y^{(1)}]_0 + \frac{x^2}{2!} \cdot 2 [y^{(2)}]_0 + \frac{x^3}{3!} \cdot 6 [y^{(1)}]_0 +$$

$$\frac{x^4}{4!} \cdot 24 (y^{(4)})_0 + \frac{x^5}{5!} \cdot 120 [y^{(5)}]_0 + \frac{x^6}{6!} = 720 [y^{(6)}]_0 +$$

$$\frac{x^7}{7!} 5040 (y^{(7)})_0$$

$$a) y (1+x)(y^{(0)})_0 + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) (y^{(1)})_0$$

$$y(0) = 0.0005 \text{ m}, y'(0) = 0.0005$$

$$y = (1+x)(0.0005 \text{ m}) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) (0.0005)$$

when  $x = 5 \text{ m}, 8 \text{ m}$  and  $10 \text{ m}$

$$y = (1+5)(0.0005 \text{ m}) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7) (0.0005)$$

$$y = 4 \times 3 \times 10^{-3} \text{ m} + 97650 \text{ m} (0.0005)$$

$$y = 3 \times 10^{-3} \text{ m} + 48.825 \text{ m}$$

$$y = 48.828 \text{ m}$$

when  $x = 8 \text{ m}$

$$y = (1+8)(0.0005) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7) (0.0005)$$

$$y = 4.5 \times 10^{-3} + 2396736 (0.0005)$$

$$y = 4.5 \times 10^{-3} + 1198$$

$$y = 1198 \text{ m}$$

When  $x = 10$

$$y = (1+10)(0.0005) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7) (0.0005)$$

$$y = 5.5 \times 10^{-3} + 1111100 (0.0005)$$

$$y = 5555.5555 \text{ m}$$

$$= 5556 \text{ m}$$

## Matlab

- (1) Command Window.
- (2) Clear
- (3) CLC
- (4) Close all
- (5) Sysms x
- 6  $x = ((1+x) * (0.0005)) + ((x^2 + x^3 + x^4 + x^5 + x^6 + x^7) *$
- 7  $(0.0005))$
- 8  $t = 0:0.01:10$
- 9  $xt = \text{subs}(x, t)$
- 10  $xt_n = \text{double}(xt)$
- 11  $\text{plot}(t, xt_n)$
- 12  $x \text{ label}('t')$
- 13  $y \text{ label}('x')$
- 14  $\text{grid on}$
- 15  $\text{grid minor}$
- 16  $\text{axis tight}$