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17/Engo3/014
Civil Engineering

1.) $x(x-1)y'' + (3x-1)y' + y = 0$

$(x^2-x)y'' + (3x-1)y' + y = 0$ Expanding bracket

we can then assume

$(x^2-x)y'' = w_1$ and $y = w_3$

$(3x-1)y' = w_2$

Using leibnitz theorem $= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$

w_1	w_2	w_3
let $u = y''$ $v = x^2 - x$	let $u = y'$ $v = 3x-1$	let $u = y$ $v = 1$
$u^n = y^{n+2}$ $v' = 2x - 1$	$u^n = y^{n+1}$ $v' = 3$	$u^n = y^n$ $v' = 0$
$u^{n-1} = y^{n+1}$ $v'' = 2$	$u^{n-1} = y^n$ $v'' = 0$	
$u^{n-2} = y^n$ $v''' = 2$		

$w_1 = (y^{n+2} \cdot (x^2-x)) + n(y^{n+1} \cdot (2x-1) + \frac{n(n-1)y^{n-2}}{2!}$

$w_2 = (y^{n+1} \cdot (3x-1) + n y^n \cdot 3 + 0$

$w_3 = y^n$

$\therefore$ $w_1 + w_2 + w_3 = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + n(n-1)y^n + y^{n+1}(3x-1) + ny^n \cdot 3$
$+ y^n = 0$

$\therefore = y^{n+2}(x^2-x) + y^{n+1}(2x-n) + y^{n+1}(3x-1) + n(n-1)y^n + ny^n \cdot 3 + y^n = 0$

assuming $x = 0$

$y^{n+1}(2(0)-n) + y^{n+1}(3(0)-1) + n(n-1)y^n + ny^n \cdot 3 + y^n = 0$

$= -ny^{n+1} - y^{n+1} + y^n(n^2-n) + 3ny^n + y^n = 0$

$= y^{n+1}(-n-1) + y^n(n^2-n+3n-1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2+2n-1) = 0$

$y^{(n+1)}(n+1) = y^n(n^2+2n-1)$

$y^{n+1}_{(n+1)} = y^n(n+1)\cancel{(n+1)}$

$y^{n+1} = y^n(n+1)$ re-occurence relation

$(y^{n+1})_0 = y^n(n+1)_0$

$(y'')_0 = 0.0005$

$(y')_0 = 0.0005$

when $n = 0$ $(y^{(n+1)})_0 = (y^{(n)})_0 (0+1)$

$(y')_0 = (y^0)_0 = 1$

when n=1 $(y^{(1+1)})_0 = (y^1)_0 (1+1)$

$(y^2)_0 = (y^1)_0\, 2 = 2(y^1)_0$

when n=2 $(y^{2+1})_0 = (y^2)_0 (2+1)$

$(y^3)_0 = 3(y^2)_0 = 6(y^1)_0$

when n=3 $(y^{3+1})_0 = (y^3)_0 (3+1)$

$(y^4)_0 = (y^3)_0\, 4 = 24 (y^1)_0$

when n=4 $(y^5)_0 = (y^4)_0\, 5 = 120 (y^1)_0$

when n=5 $(y^6)_0 = (y^5)_0\, 6 = 720 (y^1)_0$

when n=6 $(y^7)_0 = (y^6)_0\, 7 = 5040 (y^1)_0$

Using macheurin series

$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$

$y = (y^0)_0 + x(y^0)_0 + \frac{x^2}{2!}\cdot 2(y^1)_0 + \frac{x^3}{3!}\, 6(y^1)_0 + \frac{x^4}{4!}\, 24(y^1)_0 + \frac{x^5}{5!}\, 120 (y^1)_0 + \frac{x^6}{6!}\, 720 (y^1)_0$
$+ \frac{x^7}{7!}\, 5040 (y^1)_0$

$y = (y^0)_0 (x+1) + (x^2+x^3+x^4+x^5+x^6+x^7)(y^1)_0$

$\therefore y = 0.0005(x+1) + (x^2+x^3+x^4+x^5+x^6+x^7)\, 0.0005$

(ii) when $x = 5m$

$y = 0.0005(5+1) + (5^2+5^3+5^4+5^6+5^7)\, 0.0005$

$\therefore y = 48.825m$

when $x = 8m$

$y = 0.0005(8+1) + (8^2+8^3+8^4+8^5+8^6+8^7)\, 0.0005$

$\therefore y = 1198.373m$

when $x = 10$

$y = 0.0005(10+1) + (10^2+10^3+10^4+10^5+10^6+10^7)\, 0.0005$

$\therefore y = 5555.56m$

(c) MATLAB mfile

Command window

Clear

Clc

Close all

```
x = 0.001:10
y = (00005*(x+1)) + ((x^2 + x^3 + x^4 + x^5 + x^6 + x^7) * (0.0057))
yn = subs(y)
    Plot (x, yn)
    axis tight
    grid on
    grid minor
    x label ('m')
    Y label ('Deflection')
```

Sketch

Graph Of structured Element Against x