1a. $x(x-1)y + (3x-1)y + y = 0$

Applying leibnitz - maclaurin method

$(x^2-x)y'' + (3x-1)y' + y = 0$

$(x^2-x)y^2 + (3x-1)y' + y = 0$.

Find the nth derivative using leibnitz theorem

let $(x^2-x)y^{(2)} = w_1$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \dots$$

$u = y^2$. $\qquad V = x^2 - x$.

$u^n = y^{(n+2)}$ $\qquad V' = 2x - 1$

$u^{n-1} = y^{(n+1)}$ $\qquad V'' = 2$

$u^{n-2} = y^n$ $\qquad V''' = 0$

$$w_1^{(n)} = y^{(n+2)} \times (x^2 - x) + n y^{(n+1)} \times (2x-1) + \frac{n(n-1)}{2!} y^n \times 2$$

let $w_2 = (3x-1)y^{(1)}$

$u = y^{(1)}$ $\qquad V = 3x-1$

$u^{(n)} = y^{(n+1)}$ $\qquad V' = 3$

$u^{(n-1)} = y^n$ $\qquad V'' = 0$

$$w_2^{(n)} = y^{(n+1)} \times (3x-1) + n y^{(n)} \times 3$$

let $w_3 = y$

$w_3^{(n)} = y^{(n)}$

$\therefore$ nth derivative

$$y^{(n+2)} = (x^2-x) + n y^{(n+1)} \times (2x-1) + n(n-1) y^{(n)} + y^{(n+1)} \times (3x-1) + n y^{(n)} \times 3 + y^n = 0$$

Assuming $x = 0$

$n(n-1)y^{(n)} + 3n y^{(n)} + y^{(n)} - n y^{(n+1)} - y^{(n+1)} = 0$

$(n^2+2n+1)y^{(n)} - (n+1)y^{(n+1)} = 0$

$$y^{(n+1)} = \frac{(n^2+2n+1)y^{(n)}}{(n+1)} = \frac{\cancel{(n+1)}(n+1)\, y^{(n)}}{\cancel{(n+1)}}$$

$\therefore y^{(n+1)} = (n+1) y^{(n)}$

when $n=0$; $(y^{(1)})_0 = (y^{(0)})_0$

$n=1$; $(y^{(2)})_0 = (2)(y')_0 = (2y)_0$

$n=2$; $(y^3)_0 = \cancel{(3y^2)}\ \cancel{3\times 2(y)^0}\ 3(y^{(2)})_0 = 6(y^{(0)})_0$

$n=3;\ (y^{(4)})_0 = 4(y^{(3)})_0 = 4\times 6(y^{(0)}) = (24y^0)_0$

$n=4;\ (y^{(5)}) = 5(y^{4}) = 5\times 24(y^{(0)}) = (120y^0)_0$

$n=5;\ (y^{(6)}) = 6(y^{5})_0 = 6\times 120(y^{(0)}) = (720y^0)_0$

$n=6;\ (y^{7}) = 7(y^{6})_0 = 7\times 720(y^{(0)})\ (5040y^0)_0$

Maclaurin series

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \dots$$

$$y = (y^0) + x(y^0)_0 + \frac{x^2}{2!}(2(y^0)_0) + \frac{x^3}{3!}(6y^{(0)})_0 + \frac{x^4}{4!}(24(y^{(0)})_0) + \frac{x^5}{5!}(120(y^0)_0) + \frac{x^6}{6!}(720(y^0)_0) + \frac{x^7}{7!}(5040(y^0)_0 + \dots$$

$$y = (y^0)_0 + x(y^{(0)})_0 + x^2(y)_0 + x^3(y^0)_0 + x^4(y^0)_0 + x^5(y^0)_0 + x^6(y^0)_0 + x^7(y^0)_0 + \dots$$

a. Power series; $y = (y^{(0)})_0(1+x+x^2+x^3+x^4+x^5+x^6+x^7+\dots)$.

Since $y(0) = 0.0005m$ and $y'(0) = 0.0005$,

power series; $y = 0.0005(1+x+x^2+x^3+x^4+x^5+x^6+x^7+\dots)$.

b. when $x = 5m$

$y = 0.0005(1+x+x^2+x^3+x^4+x^5+x^6+x^7+\dots)$

$= 0.0005(1+5+5^2+5^3+5^4+5^5+5^6+5^7+\dots)$

$y = 48.828m$

when $x = 8m$.

$y = 0.0005(1+x+x^2+x^3+x^4+x^5+x^6+x^7+\dots)$

$= 0.0005(1+8+8^2+8^3+8^4+8^5+8^6+8^7)$

$= 1198.3735m$

when $x = 10m$

$y = 0.0005(1+x+x^2+x^3+x^4+x^5+x^6+x^7+\dots)$

$= 0.0005(1+10+10^2+10^3+10^4+10^5+10^6+10^7+\dots)$

$\therefore = 5555.5555m$

c. command window

clear

clc

close all

x = 0:0.01:10;

```
y=0.0005*(1+x+x.^2+x.^3+x.^4+x.^5+x.^6+x.^7).
yn= subs(y).
Plot (x,yn)
x label ('m')
y label ('Deflection')
axis tight
grid on.
grid minor.
```