① $x(x-1)y'' + (3x-1)y' + y = 0$

Taking $x(x-1)y'' = w_1$

$(3x-1)y' = w_2$

$y = w_3$

Considering w_1

$u = y''$ $\qquad$ $v = x(x-1)$

$u^n = y^{(n+2)}$ $\qquad$ $v' = 2x-1$

$u^{n-1} = y^{(n+1)}$ $\qquad$ $v'' = 2$

$u^{n-2} = y^2$ $\qquad$ $v''' = 0$

w_2

$u = y'$ $\qquad$ $v = (3x-1)$

$u^n = y^{(n+1)}$ $\qquad$ $v' = 3$

$u^{n-1} = y$ $\qquad$ $v'' = 0$

w_3

$u = y$ $\qquad$ $v = 1$

$u^n = y^n$ $\qquad$ $v = 0$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v''' + \dots$$

$$w_1^n = y^{n+2}\cdot(x^2-x) + n y^{(n+1)}\cdot(2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$$

$$w_2^n = y^{(n+1)}\cdot(3x-1) + n y^n \cdot 3 + 0$$

$$w_3^n = y^n \cdot 1 + 0$$

$$w_1 + w_2 + w_3$$

$$y^{n+2}\cdot(x^2-x) + n y^{n+1}\cdot(2x-1) + \frac{n(n-1)}{2} y^n \cdot 2 + y^{(n+1)}\cdot(3x-1)$$

$$n y^n - 3 + y^n = 0$$

$(x^2-x)\,y^{(n+2)} + (2x-1)\,n\,y^{(n+1)} + (3x-1)\,y^{n+1} + n(n-1)\,y^n + 3ny^n + y^n = 0$

$(x^2-x)\,y^{(n+2)} + (2xn-n)\,y^{(n+1)} + (3x-1)\,y^{(n+1)} + (n^2-2)\,y^n + 3ny^n + y^n = 0$

$(x^2-x)\,y^{(n+2)} + (2xn+3x-n-1)\,y^{(n+1)} + (n^2-n+3n+1)\,y^n = 0$

$(x^2-x)\,y^{(n+2)} + (2xn+3x-n-1)\,y^{n+1} + (n^2+2n+1)\,y^n = 0$

$(x-1)x\,y^{n+2} + (2xn+3x-n-1)\,y^{n+1} + (n^2+2n+1)\,y^n = 0$

when $x = 0$

$(0-1)\,0\,y^{(n+2)} + (2(0)n + 3(0) - n - 1)\,y^{n+1} + (n^2+2n+1)\,y^n = 0$

$(-n-1)\,y^{n+1} + (n^2+2n+1)\,y^n = 0$

$-(n+1)(y^{n+1})_0 + (n^2+2n+1)(y^n)_0 = 0$

$(y^{(n+1)})_0 = +\dfrac{(n^2+2n+1)(y^n)_0}{+(n+1)}$

$(y^{(n+1)})_0 = \dfrac{(n+1)(n+1)}{(n+1)}(y^n)_0$

$(y^{n+1})_0 = (n+1)(y^n)_0$

when $n = 0$

$(y^{0+1})_0 = (0+1)(y^0)_0$

$(y^1)_0 = 1(y^{(0)})_0$

when $n = 1$

$(y^{1+1})_0 = (1+1)(y^{(1)})_0$

$(y^{(2)})_0 = 2(y^{(1)})_0$

When $n=2$

$$(y^{(3)}]_0 = [2+1]\,[y^{(2)}]_0$$

$$[y^{(3)}]_0 = 3(y^{(3)}]_0 = 3(2)\,[y^{(2)}]_0$$

$$[y^{(3)}]_0 = 6\,[y^{(2)}]_0$$

When $n=3$

$$(y^{(4)}]_0 = 3+1)\,[y^{(3)}]_0$$

$$[y^{(4)}]_0 = 4[y^{(3)}]_0 = 4\,[6]\,[y^{(1)})_0$$

$$(y^{(4)})_0 = 24\,[y^{(1)}]_0$$

When $n=4$

$$[y^{(5)}]_0 = (4+1)\,[y^{(4)}]_0$$

$$[y^{(5)}]_0 = 5\,[y^{(4)}]_0 = 5\,(24)\,[y^{(1)}]_0$$

$$(y^{(5)})_0 = 120\,[y^{(1)}]_0$$

When $n=5$

$$(y^{(6)}]_0 = [5+1\,(y^{(5)}]_0$$

$$(y^{(6)}]_0 = 6\,[y^{(5)}]_0 = 6\,[120]\,(y^{(1)})_0$$

$$(y^{(6)})_0 = 720\,[y^{(1)}]_0$$

When $n=6$

$$(y^{(n)})_0 = (6+1)\,[y^{(6)}]_0$$

$$= 7\,[y^{(6)}]_0 = 7\,[720)(y^{(1)})_0$$

$$(y^{0}]_0 = 5040\,(y^{(1)}]_0$$

$$y = [y^{(1)}]_0 + x[y^{(1)}]_0 + \frac{x^2}{2!}(y^{(2)}]_0 + \frac{x^3}{3!}[y^{(3)}]_0 + \frac{x^4}{4!}(y^{4})_0$$

$$+ \frac{x^5}{5!}(y^{5})_0 + \frac{x^6}{6!}(y^{6})] + \frac{x^7}{7!}(y^{(7)}]_0$$

$$y = (y^{(0)})_0 + x[y^{(1)}]_0 + \frac{x^2}{2!}\cdot 2\,[y^{(2)}]_0 + \frac{x^3}{3!}\cdot 6\,[y^{(1)}]_0 +$$

$$\frac{x^4}{4!} \cdot 24(y^{(4)})_0 + \frac{x^5}{5!} \cdot 120[y^{(5)}]_0 + \frac{x^6}{6!} = 720[y^{(5)}]_0 + \frac{x^7}{7!} 5040(y^{(5)})_0$$

8. $y(1+x)(y^{(0)})_0 + (x^2+x^3+x^4+x^5+x^6+x^7)(y^{(1)})_0$

$y(0) = 0.0005m$, $y'(0) = 0.0005$

$y = (1+x)(0.0005m) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005)$

when $x = 5m$, $8m$ and $10m$

$y = (1+5)(0.0005m) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$

$y = 3\times10^{-3}m + 97650m(0.0005)$

$y = 3\times10^{-3}m + 48.825m$

$y = 48.828m$

when $x = 8m$

$y = (1+8)(0.0005) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$

$y = 4.5\times10^{-3} + 2396736(0.0005)$

$y = 4.5\times10^{-3} + 1198$

$y = 1198m$

when $x = 10$

$y = (1+10)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$

$y = 5.5\times10^{-3} + 11111100(0.0005)$

$y = 5555.5555m$

$= 5556m.$

Mat lab

(1) Command Window.
(2) Clear
(3) CLC
(4) Close all
(5) Syms x
6 x = ((1+x)*(0.0005)+((x^2+x^3+x^4+x^5+x^6+x^7)*
7 (0.0005))
8 t = 0:0.01:10
9 xt = subs (x,t)
10 xtn = double (xt)
11 plot (t, xtn)
12 x label ('t')
13 y label ('x')
14 grid on
15 grid minor
16 axis tight