

solution.

$$a) x(x-1)y'' + (2x-1)y' + y = 0$$

For the  $n$ th derivative,

$$x(x-1)y^{n+2} + (2x-1)y^{n+1} + n(n-1)y^n + (2nx-1)y^{n+1} + y^{n+1} = 0$$
$$x(x-1)y^{n+2} + y^{n+1}(2nx-1 + 2x-1) + y^n(n^2-n+2x+1) = 0$$

at  $x=0$ ,

$$(y^{n+1})_0 (n-1) + (y^n)_0 (n^2+2n+1) = 0$$
$$-(n+1)(y^{n+1})_0 = -(n^2+2n+1)(y^n)_0$$

$$(n+1)(y^{n+1})_0 = (n^2+2n+1)(y^n)_0$$
$$(y^{n+1})_0 = (n+1)(y^n)_0$$

at  $n=0$

$$(y^{0+1})_0 = (0+1)(y^0)_0$$

$$(y^1)_0 = (y^0)_0$$

at  $n=1$

$$(y^{1+1})_0 = (1+1)(y^1)_0$$

$$(y^2)_0 = 2(y^1)_0$$

at  $n=2$ ,

$$(y^{2+1})_0 = (2+1)(y^2)_0$$

$$(y^3)_0 = 3(y^2)_0$$

$$(y^3)_0 = 6(y^1)_0$$

at  $n=3$

$$(y^{3+1})_0 = (3+1)(y^3)_0$$

$$(y^4)_0 = 4 \times 6 (y^1)_0 = 24(y^1)_0$$

at  $n=4$ .

$$(y^{4+1})_0 = (4+1)(y^4)_0$$

$$= 5(y^4)_0$$

$$= 5 \times 24 (y^1)_0$$

$$(y^5)_0 = 120 (y^1)_0$$

akt  $n=5$ ,

$$C_j^i = (5+i) C_j^i$$

$$= 6 C_j^i$$

$$= 6 \times 120 C_j^i$$

$$C_j^i = 720 C_j^i$$

akt  $n=6$

$$C_j^i = (6+i) C_j^i$$

$$= 7 C_j^i$$

$$= 7 \times 720 C_j^i$$

$$C_j^i = 5040 C_j^i$$

four medium series

$$y = C_j^i + x C_j^i + x^2 C_j^i + x^3 C_j^i + x^4 C_j^i + x^5 C_j^i + \dots$$

$$\therefore y = C_j^i + x C_j^i + x^2 C_j^i + x^3 C_j^i + x^4 C_j^i + x^5 C_j^i + \dots$$

$$\frac{dx}{x} (1+x) = \frac{dx}{x} (1+x) + \frac{dx}{x} (1+x) + \frac{dx}{x} (1+x) + \dots$$

$$y = C_j^i + x C_j^i + x^2 C_j^i + x^3 C_j^i + x^4 C_j^i + x^5 C_j^i + \dots$$

$$y = C_j^i + C_j^i [x + x^2 + x^3 + x^4 + x^5 + \dots]$$

$$y(1-x) = 0.0005 \text{ and } C_j^i = 0.0005$$

$$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + \dots]$$

when  $x=5n$ ,

$$y = 0.0005 + 0.0005 [1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + \dots]$$

$$y = 0.0005 + 0.0005 (7615)$$

$$\therefore y = 48.125n$$

when  $x=8n$ ,

$$y = 0.0005 + 0.0005 [1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + \dots]$$

$$y = 0.0005 + 0.0005 (2356744)$$

$$\therefore y = 1178.3725n$$

