Assignment III

- The model for the deformation (y) of a structural element is represented by the expression given in equation (1).

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ & $y'(0) = 0.0005$, applying Leibnitz Maclaurin method;

a) Obtain the power series solution of the model up to & including the term in x^7.

b) Estimate the approximate deformation when $x = 5$, 8 & 10m

c) With the aid of a MATLAB m-file program, plot the response of the structural element for $0 \le x \le 10$ m.

Solution

a) $x(x-1)y'' + (3x-1)y' + y = 0$

- $y(0) = 0.0005$ ....., $x^7 = \ldots$

$\Rightarrow xy''(x-1) + (3x-1)y' + y = 0$.

$= y''x(x-1) - w_1 \quad y'(3x-1) - w_2 \quad , y - w_3$.

$\underline{w_1} - y''x(x-1) \Rightarrow y^{(2)} \quad x^2 - x$

$u = y^{(2)} \qquad v = x^2 - x$

$u^{(n)} = y^{(n+2)} \qquad v^{(1)} = 2x - 1$

$u^{(n-1)} = y^{(n+1)} \qquad v^{(2)}$

$w_2 - (3x-1)y^{(1)}$

$u = y^{(1)}$ $\quad v = 3x-1$

$u^{(n)} = y^{(n-1)}$ $\quad v' = 3$

$u^{(n-1)} = y^{(n)}$ $\quad v'' = 0$

$w_3 - y$

$u = y$ $\quad v = 1$

$u^{(n)} = y^{(n)}$ $\quad v' = 0$

Applying Leibnitz;

$y^{(n)} = u^{(n)}v + n u^{(n-1)}v' + \frac{n(n-1)}{2!} u^{(n-2)}v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)}v^{(3)} + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)}v^{(4)} + \ldots$

Substituting;

$w_1 = y^{(n+2)}(x^2-x) + ny^{(n+1)}(2x-1) + \frac{n(n-1)}{2!}y^{(n)}(2) + 0$

$w_2 = y^{(n+1)}(3x-1) + ny^{(n)}3 + 0$

$w_3 = y^{(n)} - 1 + 0$

$\Rightarrow y^{(n)} \Rightarrow y^{(n+2)}x(x-1) + ny^{(n+1)}(2x-1) + n(n-1)y^{(n)} + y^{(n+1)} \ldots$
$(3x-1) + ny^{(n)}(3) + y^{(n)} = 0$

$\Rightarrow y^{(n+2)}(x^2-x) + ny^{(n+1)}(2x-1) + n(n-1)y^{(n)} + y^{(n+1)}(3x-1) + \ldots$
$\ldots + 3ny^{(n)} + y^{(n)} = 0$

$\Rightarrow y^{(n+2)}(x^2-x) + (2xn-n)y^{(n+1)} + (n^2-n)y^{(n)} + (3x-1)y^{(n+1)} + \ldots$
$\ldots + 3ny^{(n)} + y^{(n)} = 0$

$\Rightarrow y^{(n+2)}(x^2-x) + (2xn-n)y^{(n+1)} + (3x-1)y^{(n+1)} + (n^2-n)y^{(n)} + \ldots$
$\ldots + 3ny^{(n)} + y^{(n)} = 0$

$\Rightarrow y^{(n+2)}(x^2-x) + y^{(n+1)}[2xn+3x-n-1] + y^{(n)}[n^2-n+3n+1] = 0$

$\Rightarrow y^{(n+2)}(x^2-x) + y^{(n+1)}[2xn+3x-n-1] + y^{(n)}[n^2+2n+1] = 0$

$n=4 \quad y^{(4+1)} = \frac{4^2+2(4)+1}{(4+1)} y^{(4)}$

$= 5y^{(4)} = (5)(24)y^{(1)}$

$y^{5} = 120y^{(1)}$

$n=5 \quad y^{(5+1)} = \frac{5^2+2(5)+1}{(5+1)} y^{(5)}$

$= 6y^{(5)} = (6)(120)y^{(1)}$

$y^{(6)} = 720y^{(1)}$

$n=6 \quad y^{(6+1)} = \frac{6^2+2(6)+1}{(6+1)} y^{(6)}$

$= 7y^{(6)} = (7)(720)y^{(1)}$

$y^{(7)} = 5040y^{(1)}$

Applying Maclaurin's theorem

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 + \cdots$

$\cdots + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0 + \cdots$

Substituting;

$y = y^{(0)} + xy^{(0)} + \frac{x^2}{2!}2y^{(1)} + \frac{x^3}{3!}6y^{(1)} + \frac{x^4}{4!}24y^{(1)} + \cdots$

$\cdots + \frac{x^5}{5!}120y^{(1)} + \frac{x^6}{6!}720y^{(1)} + \frac{x^7}{7!}5040y^{(1)} + \cdots$

$y = y^{(0)}(1+x) + y^{(1)}\left[\frac{2x^2}{2!} + \frac{6x^3}{3!} + \frac{24x^4}{4!} + \frac{120x^5}{5!} + \cdots \right.$

$\left. \cdots + \frac{720x^6}{6!} + \frac{5040x^7}{7!} + \cdots\right]$

$\therefore$

When $x=0$

$$\Rightarrow \cancel{y^{(n+2)}(0-0)} + y^{(n+1)}(\cancel{2m+3m} - n-1) + y^{(n)}(n^2+2n+1) = 0$$

$$\Rightarrow (y^{(n+1)})_0(-n-1) + (y^{(n)})_0(n^2+2n+1)$$

$$= -(n+1)(y^{(n+1)})_0 = -(n^2+2n+1)(y^{(n)})_0$$

$$(y^{(n+1)})_0 = \frac{+(n^2+2n+1)}{+(n+1)}(y^{(n)})_0$$

when $n=0$

$$y^{(0+1)} = \frac{0^2+2(0)+1}{(0+1)} y^{(0)}$$

$$y^{(1)} = 1y^{(0)}$$

$n=1$ $$y^{(1+1)} = \frac{1^2+2(1)+1}{(1+1)} y^{(1)}$$

$$y^{(2)} = 2y^{(1)}$$

$n=2$ $$y^{(2+1)} = \frac{2^2+2(2)+1}{(2+1)} y^{(2)}$$

$$= 3y^{(2)} = (3)(2)y^{(1)}$$

$$y^{(3)} = 6y^{(1)}$$

$n=3$ $$y^{(3+1)} = \frac{3^2+2(3)+1}{(3+1)} y^{(3)}$$

$$= 4y^{(3)} = (4)(6)y^{(1)}$$

$$y^{(4)} = 24\,y^{(1)}$$

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close all
syms x
x = [(1+x)+(0.0005)]+[(x^2 + x^3 + x^4 + x^5+x
t = 0:0.01:10
xt = subs (x,t)
xtn = double (xt)
plot (t,xtn)
xlabel ('t')
ylabel ('x')
grid on
grid minor
axis tight
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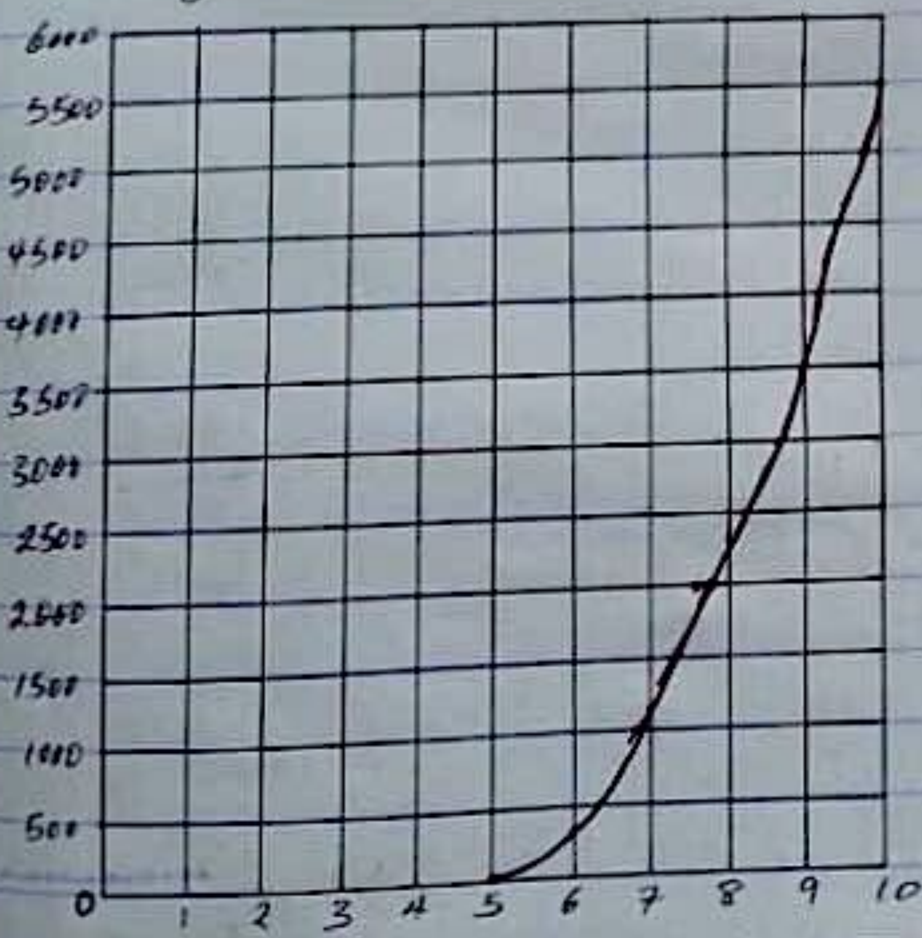

b.) $y_{(0)} = 0.0005m$, $y^{(1)}_{(0)} = 0.0005$

deformation when $x = 5, 8$ & 10

$\therefore y = (1+x)(y^{(0)}) + (x^2+x^3+x^4+x^5+x^6+x^7)(y^{(1)})$.

$\Rightarrow y = (1+x)(0.0005m) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005m)$

when $x = 5$

$y = (1+5)(0.0005) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$

$= 0.003 + 48.825$

$y = 48.828\,m$ //

when $x = 8$

$y = (1+8)(0.0005) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$

$= 0.0045 + 1198.368$

$y = 1198.3725\,m$ // $\simeq 1198.37m$ //

when $x = 10$

$y = (1+10)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$

$= 0.0055 + 5555.55$

$y = 5555.5555m$

$y \simeq 5555.56m$ //

(c) MATLAB

response for $0 \le x \le 10m$.