1. Solution

(a) $x(x-1)y'' + (3x-1)y' + y = 0$

for the nth derivative,

$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$

$x(x-1)y^{n+2} + y^{n+1}(2xn - n + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$

at $x = 0$,

$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 + 2n + 1) = 0$

$-(n+1)(y^{n+1})_0 = -(n^2 + 2n + 1)(y^n)_0$

$(n+1)(y^{n+1})_0 = (n^2 + 2n + 1)(y^n)_0$

$(y^{n+1})_0 = (n+1)(y^n)_0$

at $n = 0$,

$(y^{0+1})_0 = (0+1)(y^0)_0$

$(y')_0 = (y)_0$

at $n = 1$,

$(y^{1+1})_0 = (1+1)(y')_0$

$(y^2)_0 = 2(y')_0$

at $n = 2$,

$(y^{2+1})_0 = (2+1)(y^2)_0$

$(y^3)_0 = 3(y^2)_0$

$(y^3)_0 = 6(y')_0$