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17/ENG05/1024
MECHATRONICS ENGR

$x(x-1)y'' + (3x-1)y' + y = 0$

Let $w_1 = x(x-1)y'' = (x^2-x)y''$

$\therefore v = (x^2-x)$, $v^{(1)} = 2x-1$, $v^{(2)} = 2$, $v^{(3)} = 0$

$u = y''$, $u' = y'''$ $\therefore u^n = y^{n+2}$

$w_1^n = u^n v + n u^{n-1} v^{(1)} + \frac{n(n-1)}{2} u^{n-2} v^{(2)} + \frac{n(n-1)(n-2)}{3\cdot 2} u^{n-3} v^{(3)}$

$= y^{n+2}(x^2-x) + n y^{n+1}(2x-1) + \frac{n(n-1)}{2} y^n \cdot 2 + \cancel{\frac{n(n-1)(n-2)}{3\cdot 2} y^{n-1}(0)}$

$= (x^2-x)y^{n+2} + n(2x-1)y^{n+1} + (n^2+n)y^n$

Let $w_2 = (3x-1)y'$

$\therefore v = 3x-1$, $v^{(1)} = 3$, $v^{(2)} = 0$

$u = y'$ $\therefore u^n = y^{n+1}$

$w_2^n = u^n v + n u^{n-1} v^{(1)} + \frac{n(n-1)}{2} u^{n-2} v^{(2)}$

$= y^{n+1}(3x-1) + n y^n \cdot 3 + \cancel{\frac{n(n-1)}{2} y^{n-1}(0)}$

$= (3x-1)y^{n+1} + 3n y^n$

Let $w_3 = y$

$\therefore u = y$ $\therefore u^n = y^n$

$v = 1$, $v^{(1)} = 0$

$w_3^n = u^n v + n u^{n-1} v^{(1)}$

$= y^n \cdot 1 + \cancel{n y^{n-1}(0)}$

$= y^n$

$\therefore w_1^n + w_2^n + w_3^n$

$\Rightarrow (x^2-x)y^{n+2} + n(2x-1)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3n y^n + y^n$

$= (x^2-x)y^{n+2} + [n(2x-1) + (3x-1)]y^{n+1} + (n^2+2n+1)y^n$

$\therefore$ at $x = 0$

$\Rightarrow 0 \cdot y_0^{n+2} + (-n-1)y_0^{n+1} + (n^2+2n+1)y_0^n = 0$

$-(n+1)y_0^{n+1} + (n^2+2n+1)y_0^n = 0$

$y_0^{n+1} = \frac{(n^2+2n+1)}{n+1} y_0^n$

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$$y_0^{n+1} = \frac{(n+1)(n+1)\,y_0^n}{(n+1)}$$

$$y_0^{n+1} = (n+1)\,y_0^n$$

at $n = 1 \Rightarrow y_0^{(2)} = 2\,y_0^{(1)}$

at $n = 2 \Rightarrow y_0^{(3)} = 3\,y_0^{(2)} = 3 \times 2\,y_0^{(1)}$

at $n = 3 \Rightarrow y_0^{(4)} = 4\,y_0^{(3)} = 4 \times 3 \times 2\,y_0^{(1)}$

at $n = 4 \Rightarrow y_0^{(5)} = 5\,y_0^{(4)} = 5 \times 4 \times 3 \times 2\,y_0^{(1)}$

at $n = 5 \Rightarrow y_0^{(6)} = 6\,y_0^{(5)} = 6 \times 5 \times 4 \times 3 \times 2\,y_0^{(1)}$

at $n = 6 \Rightarrow y_0^{(7)} = 7\,y_0^{(6)} = 7 \times 6 \times 5 \times 4 \times 3 \times 2\,y_0^{(1)}$

maclaurin series: $\Rightarrow y = y_0 + x\,y_0^{(1)} + \frac{x^2}{2!}\,y_0^{(2)} + \frac{x^3}{3!}\,y_0^{(3)} + \frac{x^4}{4!}\,y_0^{(4)} + \frac{x^5}{5!}\,y_0^{(5)} + \frac{x^6}{6!}\,y_0^{(6)} + \frac{x^7}{7!}\,y_0^{(7)}$

$$\therefore y = y_0 + x\,y_0^{(1)} + \frac{x^2}{2} \times 2\,y_0^{(1)} + \frac{x^3}{3 \cdot 2} \times 3 \cdot 2\,y_0^{(1)} + \frac{x^4}{4 \cdot 3 \cdot 2} \times 4 \cdot 3 \cdot 2\,y_0^{(1)} + \frac{x^5}{5 \cdot 4 \cdot 3 \cdot 2} \times 5 \cdot 4 \cdot 3 \cdot 2\,y_0^{(1)} + \frac{x^6}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \times 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2\,y_0^{(1)} + \frac{x^7}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \times 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2\,y_0^{(1)}$$

$$y = y_0 + x\,y_0^{(1)} + x^2 y_0^{(1)} + x^3 y_0^{(1)} + x^4 y_0^{(1)} + x^5 y_0^{(1)} + x^6 y_0^{(1)} + x^7 y_0^{(1)}$$

$$y = y_0 + y_0^{(1)}\,(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7).$$

since $y_0 = 0.0005m$ & $y_0^{(1)} = 0.0005$

$$\therefore y = 0.0005 + 0.0005\,(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

(b) when $x = 5m$

$$y = 0.0005 + 0.0005\,(5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$$

$$y = 48.828$$

when $x = 8m$

$$y = 0.0005 + 0.0005\,(8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$$

$$y = 1198.3725$$

when $x = 10m$

$$y = 0.0005 + 0.0005\,(10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$$

$$y = 5555.55$$

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```
clear
clc
close all
x = 0:10
Y = 0.0005 + (0.0005 * (x + x.^2 + x.^3 + x.^4 + x.^5 + x.^6 + x.^7))
Plot (x, Y)
Xlabel ('Length')
Ylabel ('Deformation')
title ('Length - deformation graph')
grid on
grid minor
axis tight
```

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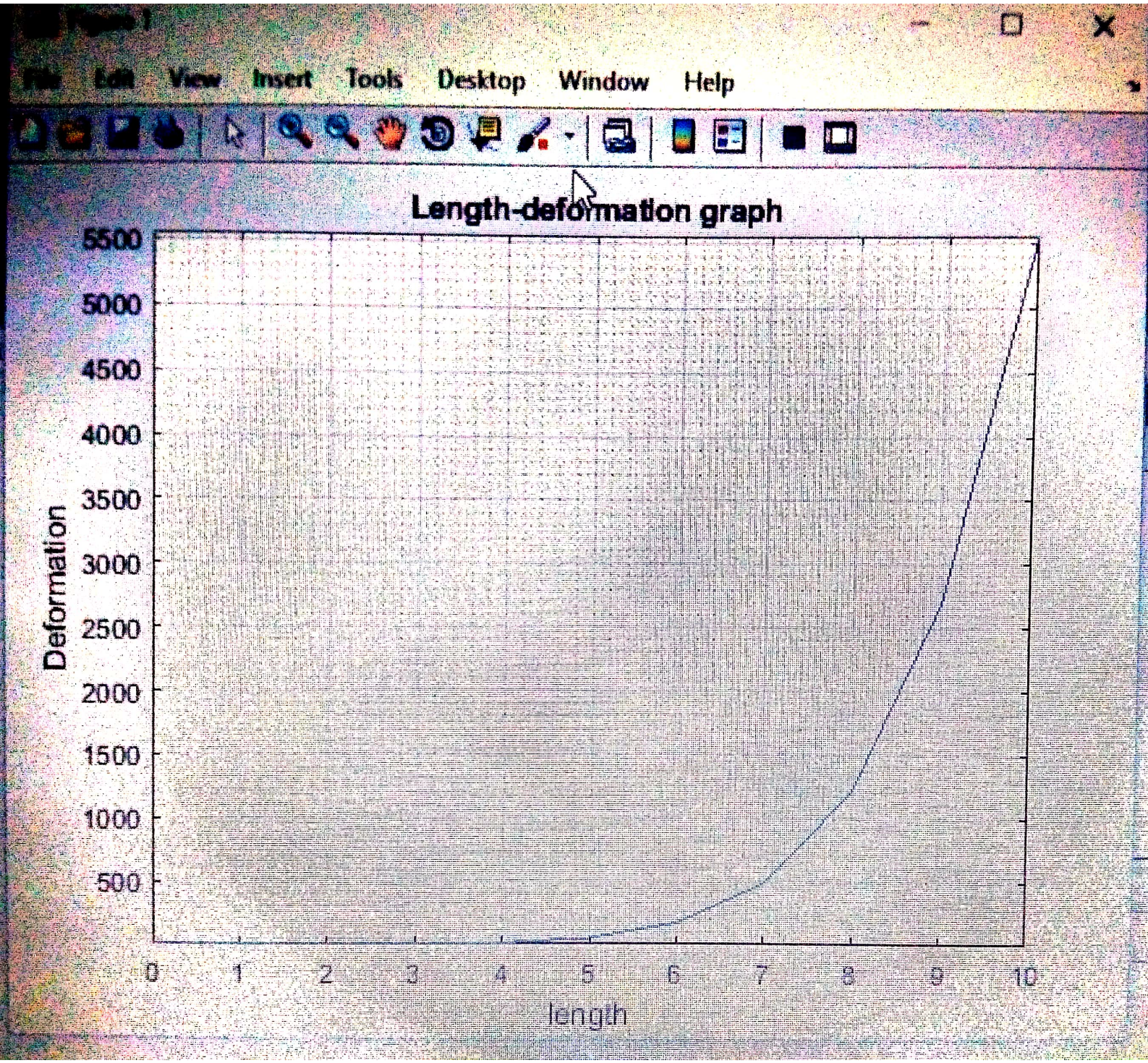

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