

Solution
 (a) $x(x-1)y'' + (2x-1)y' + y = 0$
 for the n th derivative,
 $x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (2x-1)y^{n+1} + 2ny^{n+1} + y^n$
 $x(x-1)y^{n+2} + y^{n+1}(2xn-n+2x-1) + y^n(n-n+2n+1) = 0$
 at $x=0$,
 $(y^{n+2})_0(-n-1) + (y^n)_0(n^2+2n+1) = 0$
 $-(n+1)(y^{n+2})_0 = -(n^2+2n+1)(y^n)_0$
 $(y^{n+2})_0 = \frac{(n^2+2n+1)(y^n)_0}{(n+1)}$
 at $n=0$,
 $(y^{2})_0 = (0+1)(y^0)_0$
 $(y')_0 = (y)_0$
 at $n=1$,
 $(y^{3})_0 = (1+1)(y^1)_0$
 $(y^2)_0 = 2(y')_0$
 at $n=2$,
 $(y^{4})_0 = (2+1)(y^2)_0$
 $(y^3)_0 = 3(y^2)_0$
 $(y^4)_0 = 6(y^3)_0$

$(y^5)_0 = (4+1)(y^4)_0$
 $(y^6)_0 = (5+1)(y^5)_0 = 6 \cdot 5 (y^4)_0 = 30 (y^4)_0$
 $(y^7)_0 = (6+1)(y^6)_0 = 7 \cdot 30 (y^4)_0 = 210 (y^4)_0$
 $(y^8)_0 = (7+1)(y^7)_0 = 8 \cdot 210 (y^4)_0 = 1680 (y^4)_0$
 $(y^9)_0 = (8+1)(y^8)_0 = 9 \cdot 1680 (y^4)_0 = 15120 (y^4)_0$
 $(y^{10})_0 = (9+1)(y^9)_0 = 10 \cdot 15120 (y^4)_0 = 151200 (y^4)_0$
 from Maclaurin series,
 $y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$
 $y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(2(y')_0^2) + \frac{x^3}{3!}(6(y')_0^3) + \frac{x^4}{4!}(24(y')_0^4) + \dots$
 $+ \frac{x^5}{5!}(120(y')_0^5) + \frac{x^6}{6!}(720(y')_0^6) + \frac{x^7}{7!}(5040(y')_0^7) + \dots$

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 $y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(2(y')_0^2) + \frac{x^3}{3!}(6(y')_0^3) + \frac{x^4}{4!}(24(y')_0^4) + \dots$
 $y = 0.0005 + 0.0005[x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + \dots]$
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 when $x = 5$,
 $y = 0.0005 + 0.0005[5 + 25 + 125 + 625 + 3125 + 15625 + 78125 + 390625 + 1953125 + 9765625 + 48828125 + \dots]$
 $y = 0.0005 + 0.0005[231470]$
 $y = 115.735$
 when $x = 8$,
 $y = 0.0005 + 0.0005[8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152 + 16777216 + 134217728 + 1073741824 + 8589934608 + \dots]$
 $y = 0.0005 + 0.0005[1111110]$
 $y = 555.555$

get command-line
 clear
 %u
 syms x
 eqns =
 (0^2-10);
 y = 0.0005 + 0.0005*(x + x^2) + (x^3)^2 + (x^4)^2 + (x^5)^2;
 diff(y,x)
 plot on
 grid minor
 label('x')
 format('Standard Representation')