

OGBONNA VICTOR CHIBUZO

ELECTRICAL/ELECTRONICS ENGINEERING

17/06/2018

ENGR 381

i)

$$a) x(x-1)y'' + (3x-1)y' + y = 0$$

For the  $n$ th derivative:

$$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n$$

$$x(x-1)y^{n+2} + y^{n+1}(2x^2 - x + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$$

at  $x=0$

$$(y^{n+1})(-n-1) + (y^n) = (n^2 + 2n + 1) - 0$$
$$-(n+1)(y^{n+1}) = -(n^2 + 2n + 1)(y^n)$$

$$(n+1)(y^{n+1}) = (n^2 + 2n + 1)(y^n)$$
$$(y^{n+1})_0 = (n+1)(y^n)_0$$

at  $n=0$

$$(y^{1+0})_0 = (0+1)(y^0)_0$$
$$(y^1)_0 = (y^0)_0$$

at  $n=1$

$$(y^{1+1})_0 = (1+1)(y^1)_0$$
$$(y^2)_0 = 2(y^1)_0$$

at  $n=2$

$$(y^{2+1})_0 = (2+1)(y^2)_0$$
$$(y^3)_0 = 3(y^2)_0$$
$$(y^3)_0 = 6(y^2)_0$$

$$(y^4)_0 = (3+1)(y^2)$$

$$(y^4)_0 = 4(y^2)_0 = 4 \times 6(y)_0 = 24(y')_0$$

at  $n=4$

$$(y^3)_0 = (4+1)(y^4)_0$$

$$y^3 = 5(y^4)_0 = 5 \cdot 24(y')_0$$

$$(y^3)_0 = 120(y')_0$$

at  $n=5$

$$(y^6)_0 = (5+1)(y^5)_0$$

$$6(y^5)_0$$

$$(y^6)_0 = 6 \cdot 120(y')_0$$

$$(y^6)_0 = 720(y')_0$$

at  $n=6$

$$(y^7)_0 = (6+1)(y^6)_0$$

$$(y^7)_0 = 7(y^6)_0$$

$$(y^7)_0 = 7 \cdot 720(y')_0$$

$$(y^7)_0 = 5040(y')_0$$

Using Maclaurin's series

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0$$

$$\therefore y = (y)_0 + x(y')_0 + \frac{x^2}{2!}[2(y^7)_0] + \frac{x^3}{3!}[6(y^6)_0] + \frac{x^4}{4!}[24(y^5)_0]$$

$$\Rightarrow \frac{x^5}{5!}[120(y^4)_0] + \frac{x^6}{6!}[720(y^3)_0] + \frac{x^7}{7!}[5040(y^2)_0]$$

$$y = (y)_0 + x(y')_0 + x^2(y^2)_0 + x^3(y^3)_0 + x^4(y^4)_0 + x^5(y^5)_0 + x^6(y^6)_0 + x^7(y^7)_0 + \dots$$

$$y = (y)_0 + (y')_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots]$$

$$\text{but } (y)_0 = 0.0005 \text{ and } (y')_0 = 0.0005$$

$$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

(b)

when  $x = 5m$

$$y = 0.0005 + 0.0005 [5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$

$$y = 0.0005 + 0.0005 (97655)$$

$$\therefore y = 48.428m //$$

when  $x = 8m$

$$y = 0.0005 + 0.0005 [8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152]$$

$$y = 0.0005 + 0.0005 (2396744)$$

$$\therefore y = 1198.3725 \text{ meters}$$

when  $x = 10m$

$$y = 0.0005 + 0.0005 [10 + 100 + 1000 + 10000 + 100000 + 1000000]$$

$$y = 0.0005 + 0.0005 [1111110]$$

$$y = 5555.56m //$$

(c)

Command window

clear

clc

syms x

syms y

x = (0:10)

$$y = 0.0005 + 0.0005 [x + (x \wedge 2) + (x \wedge 3) + (x \wedge 4) + (x \wedge 5) + (x \wedge 6) + (x \wedge 7)];$$

plot (x,y)

grid on

grid minor

ylabel('x')

ylabel('structural Deformation')