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17/ENG06/014
Mechatronics Engineering

Assignment 3

1a, $x(x-1)y + (3x-1)y + y = 0$

Applying leibnitz - maclaurins method

$(x^2 - x)y'' + (3x-1)y' + y = 0$

$(x^2 - x)y^2 + (3x-1)y' + y = 0$

let $(x^2 - x)y^{(2)} = \omega_1$

$$y^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \ldots$$

$u = y^2$ $\qquad v = x^2 - x$

$u^n = y^{(n+2)}$ $\qquad v' = 2x - 1$

$u^{n-1} = y^{(n+1)}$ $\qquad v'' = 2$

$u^{n-2} = y^n$ $\qquad v''' = 0$

$$\omega_1^{(n)} = y^{(n+2)} \times (x^2 - x) + ny^{(n+1)} \times (2x-1) + \frac{n(n-1)}{2!}y^n \times 2$$

let $\omega_2 = (3x-1)y^{(1)}$

$u = y^{(1)}$ $\quad u^{(n)} = y^{(n+1)}$ $\quad u^{(n-1)} = y^n$

$V = 3x - 1$ $\quad v' = 3$ $\quad v'' = 0$

$$\omega_2^{(n)} = y^{(n+1)} \times (3x-1) + ny^{(n)} \times 3$$

let $\omega_3 = y$

$\omega_3^{(n)} = y^{(n)}$

$\therefore$ nth derivative

$$y^{(n+2)} = (x^2 - x) + ny^{(+1)} \times (2x-1) + n(n-1)y^{(n)} + y^{(n+1)} \times (3x-1) + ny^{(n)} \times 3 + y^n = 0$$

Assuming $x = 0$

$$n(n-1)y^{(n)} + 3ny^{(n)} + y^{(n)} - ny^{(n+1)} - y^{(n+1)} = 0$$

$$(n^2 + 2n + 1)y^{(n)} - (n+1)y^{(n+1)} = 0$$

$$y^{(n+1)} = \frac{(n^2 + 2n + 1)}{(n+1)}y^{(n)} = \frac{(n+1)(n+1)}{(n+1)}y^{(n)}$$

$$\therefore y^{(n+1)} = (n+1)y^{(n)}$$

When $n = 0 \rightarrow (y^{(1)})_0 = (y^{(0)})_0$

$n = 1 \rightarrow (y^{(2)})_0 = (2)(y')_0 = (2y)_0$

$n=2 \rightarrow (y^3)_0 = 3(y^{(2)})_0 = 6(y^{(0)})_0$

$n=3 \rightarrow (y^{(4)})_0 = 4(y^{(3)})_0 = 4\times6(y^{(0)}) = (24y^0)_0$

$n=4 \rightarrow (y^{(5)}) = 5(y^4) = 5\times24(y^{(0)}) = (120y^0)_0$

$n=5 \rightarrow (y^{(6)}) = 6(y^5)_0 = 6\times120(y^{(0)}) = (720y^0)_0$

$n=6 \rightarrow (y^{(7)}) = 7(y^6)_0 = 7\times720(y^{(0)}) = (5040y^0)_0$

Maclaurin Series

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \dots$$

$$y = (y^0)_0 + x(y^0)_0 + \frac{x^2}{2!}(2(y^0)_0) + \frac{x^3}{3!}(6y^{(0)})_0 + \frac{x^4}{4!}(24(y^{(0)})_0) + \frac{x^5}{5!}(120(y^0)_0) + \frac{x^6}{6!}(720(y^0)_0 + \frac{x^7}{7!}(5040(y^0)_0 + \dots$$

$$y = (y^0)_0 + x(y^{(0)})_0 + x^2(y^0)_0 + x^3(y^0)_0 + x^4(y^0)_0 + x^5(y^0)_0 + x^6(y^0)_0 + x^7(y^0)_0 + \dots$$

a, Power Series

$$y = (y^{(0)})_0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots)$$

Since $y(0) = 0.0005m$ & $y(0) = 0.0005$

$$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots)$$

b, When $x = 5m$

$$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots)$$

$$= 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7 + \dots)$$

$$= 48.83m$$

When $x = 8m$

$$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots)$$

$$= 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7 + \dots)$$

$$= 1198.37m$$

When $x = 10m$

$$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots)$$

$$= 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7 + \dots)$$

$$= 5555.56m$$

C) Command Window

```
Clear
Clc
Close all
x =0: 0.01 : 10
y= 0.0005* (1+x +x^2 +x^3 + x^4 +x^5 + x^6 + x^7)
yn= Subs (y)
Plot (x, yn)
x label ('m')
y label ('Deflection')
axis tight
grid on
grid minor
```

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