MEJU CHIAGOZIEM
17/Sci01/089
MECHATRONICS

1a. $x(x-1)y + (3x-1)y + y = 0$

Applying Leibnitz-maclaurin method

$(x^2-x)y'' + (3x-1)y' + y = 0$

$(x^2-x)y^2 + (3x-1)y' + y = 0$

Find the nth derivative using Leibnitz

let $(x^2-x)y^{(2)} = \omega$

$y^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' +$

$u = y^2 \qquad V = x^2 - x$

$u^n = y^{(n+2)} \qquad V' = 2x - 1$

$u^{(n-1)} = y^{(n+1)} \qquad V'' = 2$

$u^{(n-2)} = y^n \qquad V''' = 0$

$\omega_1^{(n)} = y^{(n+2)} \cdot (x^2-x) + ny^{(n+1)}(2x-1) + \frac{n(n-1)}{2!}y^n + 2$

let $\omega_2 = (3x-1)y''$

$u = y^{(1)} \qquad V = 3x-1$

$u^n = y^{(n+1)} \qquad V' = 3$

$u^{n-1} = y^n \qquad V^2 = 0$

For ω_3 :

$u = y \qquad V = 1$

$u^n = y^n \qquad V' = 0$

$\omega_1 = y^{(n+2)}(x^2-x) + ny^{(n+1)}(2x-1) + n(n-1)y^n \cdot 2$

$= (x^2-x)y^{(n+2)} + n(2x-1)y^{(n+1)} + n(n-1)y^n$

$\omega_2 = y^{(n+1)}(3x\mp 1) + 3ny^n$

$\omega_3 = y^n$

$$y+\omega_2+\omega_3 = (x^2-x)y^{(n+2)} + n(2x-1)y^{(n+1)} + (n^2-n)y^n + y^{(n+1)}(3x-1) + 3xy^n + y^n$$

assigning $x=0$

$$(0-0)y^{(n+2)} + (2n(0)-n)y^{n+1} + (n^2-n)y^n + (3(0)-1)y^{(n+1)} + 3y^n + y^n = 0$$

$$-ny^{(n+1)} + (n^2-n)y^n - y^{(n+1)} + 3ny^n + y^n = 0$$

Collecting like terms

$$y^{(n+1)}(-n-1) + y^n(n^2-n+3n+1) = 0$$

$$-y^{n+1}(n+1) + y^n(n^2+2n+1) = 0$$

$$(n+1)y^{(n+1)} = (n^2+2n+1)y^n \quad [(n^2+2n+1) = (n+1)(n+2)]$$

$$(n+1)y^{(n+1)} = (n+1)(n+2)y^n$$

Divide both sides by $(n+1)$

$$y^{(n+1)} = (n+1)y^n \quad \text{- Recurrence.}$$

$$y^{(n+1)}_0 = y^n(n+1)$$

$$y^0(y^1)_0 = 0 \cdot 005$$

when $n=0$

$$[y^{(0+1)}]_0 = (0+1)(y^0)$$

$$y^{(1)}_0 = 1\,[y^0]_0$$

$$n=1;\ [y^{(1+1)}]_0 = (1+1)(y^1)_0$$

$$(y^{(2)})_0 = 2(y^1)$$

$$n=2;\ (y^3)_0 = (2+1)y^2 = 3[y^2] = 3[2(y^{(1)})]_0 = 6y'_0$$

$$n=3;\ (y^4)_0 = (3+1)y^3 = 4(y^3) = 4[6(y')] = 24(y')_0$$

$$n=4;\ (y^5)_0 = (4+1)y^4 = 5(y^4)_0 = 5(24(y')_0) = 120(y_0)$$

$$n=5;\ (y^6)_0 = (5+1)y^5 = 6(y^5)_0 = 6[120(y')] = 720(y')_0$$

$$n=6;\ (y^7) = (6+1)y^6 = 7y^6 = 7[720(y')_0] = 5040(y')_0$$

Using Maclaurin's theory:

$$y = (y^0_0)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3) + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5) + \frac{x^6}{6}(y^6)_0 + \frac{x^7}{7!}(y^7)$$

= [illegible]

$$Y = (y')_0 + x(y') + \frac{x^2}{2!}(2y')_0 + \frac{x^3}{3!}(6y') + \frac{x^4}{4!}(24y') + \frac{x^5}{5!}(120y') + \frac{x^6}{6!}(720y')$$

$$\frac{x^7}{7}(5040y')$$

$$Y = Y'(1+x) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)y'$$

$$= 0.005(1+x) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)0.005$$

ii) Estimate the approximate deformation when $x = 5, 8$ and $10m$.

when $x = 5m$

$$Y = Y'(1+5) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)\ 0.005$$

$$= 0.005(6) + (25 + 125 + 625 + 3125 + 15625 + 78125)\ 0.005$$

$$= 488.28m$$

when $x = 8m$

$$Y = Y'(1+8) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)y'$$

$$= 0.005(1+8) + (64 + 512 + 4096 + 32764 + 262144 + 2097152)0.005$$

$$= 11198.3725m$$

when $x = 10m$

$$Y = Y'(1+10) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)y'$$

$$= 0.005(11) + (100 + 1000 + 10000 + 100000 + 1000000 + 10000000)\ 0.005$$

$$= 55555.56m$$

matlab mfile

Command window

```
clear
clc
close all
x = 0:0.01:10
y = (0.005 * (1+x)) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) * 0.005
yn = subs (y)
Plot (x, yn)
```

```
x label (x, y2)
x label ('m')
y label ('deflection')
axis tight
grid on
grid minor
```