

Uttobang, Odotekom Peter

Civl Engineering

17/ENG03/CE2

ENG 381 - Engineering Mathematics III



Assignment:

The model for the deformation (y) of a structural element is represented by the expression given in equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005$ and $y'(0) = 0.0005$, applying Leibnitz-Mclaurin method

- Obtain the power series solution of the model up to and including the term in x^4 .
- estimate the appropriate deformation when $x = 5, 8$ and 10
- With the aid of a matlab mfile program, plot the response of the structural element for $0 \leq x \leq 10m$.

Solution

Using Leibnitz Method

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + \frac{n(n-1)}{2} 2y^n + (3x-1)y^{n+1} + n(3)y^n + y^n = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n)y^{n+1} + (n^2-n)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2-n+3n+1)y^n = 0$$

$$(x^2-x)y^{n+2} + (2nx-n+3x-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

When $x=0$

$$(0^2-0)y^{n+2} + (2n(0)-n+3(0)-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$-(n+1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$y^{n+1} = \frac{(n^2+2n+1)}{(n+1)} y^n$$

$$y_0^{n+1} = \frac{(n+1)(n+1)}{(n+1)} y_0^n$$

$$y_0^{n+1} = (n+1)y_0^n$$

$$\begin{aligned}
n=0; y^1 &= 1y_0' \\
n=1; y^2 &= 2y_0' \\
n=2; y^3 &= 2(2)y_0' = 6y_0' \\
n=3; y^4 &= 4(3)(2)y_0' = 24(y_0') \\
n=4; y^5 &= 5(4)(3)(2)y_0' = 5!y_0' \\
n=5; y^6 &= 6y_0^5 = 6(5)(4)(3)(2)y_0' = 6!y_0' \\
n=6; y^7 &= 7y_0^6 = 7(6)(5)(4)(3)(2)y_0' = 7!y_0'
\end{aligned}$$

Maclaurin Series

$$\begin{aligned}
y &= y_0 + x(y_0') + \frac{x^2}{2!}(y_0'') + \frac{x^3}{3!}(y_0''') + \frac{x^4}{4!}(y_0^{(4)}) + \frac{x^5}{5!}(y_0^{(5)}) + \frac{x^6}{6!}(y_0^{(6)}) + \frac{x^7}{7!}(y_0^{(7)}) \\
&= \frac{y_0^0 + x(y_0') + \frac{x^2}{2!}(2y_0') + \frac{x^3}{3!}(3!y_0') + \frac{x^4}{4!}(4!y_0') + \frac{x^5}{5!}(5!y_0') + \frac{x^6}{6!}(6!y_0') + \frac{x^7}{7!}(7!y_0')}{1} \\
&= y_0^0 + x(y_0') + x^2(y_0') + x^3(y_0') + x^4(y_0') + x^5(y_0') + x^6(y_0') + x^7(y_0') \\
&= y_0^0 + y_0'(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)
\end{aligned}$$

Recall

$$\begin{aligned}
y^1 &= y_0' \\
y_0 &(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7) \\
y^1 &= 0.0005 \text{ and } y^0 = 0.0005, \text{ when } x = 5 \\
y^5 &= 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7) \\
&= 0.0005(97656) \\
&= 48.828 \\
&\approx 49.
\end{aligned}$$

$$\text{when } x = 8, y_0^0 = 0.0005$$

$$\begin{aligned}
y_8 &= 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7) \\
&= 0.0005(23967431) \\
&= 1198.3725 \\
&\approx 1198
\end{aligned}$$

$$x = 10$$

$$y_{10} = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$$

$$= 0.0005 (11111111)$$

$$\approx 5555.5555$$

$$\approx 5556.$$

© Command window

clc

clear all

close all

syms x, y

$$x = 0:0:1:10$$

$$y = (0.0005)^x (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

$$y_n = \text{subs}(y)$$

$$y_{nn} = \text{double}(y_n)$$

Plot (x, y_{nn})

x label ('x')

y label ('T')

grid on

grid minor

axis right.