

17/ENG902/066

ASSIGNMENT III

1)  $x(x-1)y'' + (3x-1)y' + y = 0$

Taking  $x(x-1)y'' = w_1$   
 $(3x-1)y' = w_2$   
 $y = w_3$

Considering  $w_1$   
 $U = y''$   
 $U'' = y^{(n+2)}$   
 $U^{n+1} = y^{(n+1)}$   
 $U^{n+2} = y''$

$V = x(x-1)$   
 $V' = 2x-1$   
 $V'' = 2$   
 $V''' = 0$

$w_2$   
 $U = y'$   
 $U'' = y^{(n+1)}$   
 $U^{n+1} = y$

$V = (3x-1)$   
 $V' = 3$   
 $V'' = 0$

$w_3$   
 $U = y$   
 $U'' = y''$

$V = 1$   
 $V' = 0$

$y'' = U''V + nU'V' + \frac{n(n-1)}{2!}U^{(n-2)}V'' + n\frac{(n-1)(n-2)}{3!}U^{(n-3)}V''' + \dots$

$w_1'' = y^{(n+2)}, (3x-1)y^{(n+2)} + n(3x-1)y^{(n+1)} + \frac{n(n-1)}{2!}y^{(n)} + \frac{n(n-1)(n-2)}{3!}y^{(n-1)} + \dots = 0$

$w_2'' = y^{(n+1)}, (3x-1)y^{(n+1)} + n(3x-1)y^{(n)} + n \cdot 3y^{(n-1)} + \dots = 0$

$w_3'' = y'' + 0 + 0 + \dots = 0$

$y^{(n+2)} + (3x-1)y^{(n+1)} + n(3x-1)y^{(n)} + n \cdot 3y^{(n-1)} + \dots = 0$

$ny'' + 3y' = 0$

$$(x^2 - x) y^{(n+2)} + (2x-1) n y^{(n+1)} + (3x-1) y^{(n+1)} + n(n-1) y^n + 3n y^n + y^n = 0$$

$$(x^2 - x) y^{(n+2)} + (2xn - n) y^{(n+1)} + (3x-1) y^{(n+1)} + (n^2 - 2) y^n + 3n y^n + y^n = 0$$

$$(x^2 - x) y^{(n+2)} + (2xn + 3x - n - 1) y^{(n+1)} + (n^2 - n + 3n + 1) y^n = 0$$

$$(x^2 - x) y^{(n+2)} + (2xn + 3x - n - 1) y^{(n+1)} + (n^2 + 2n + 1) y^n = 0$$

$$(x-1) x y^{(n+2)} + (2xn + 3x - n - 1) y^{(n+1)} + (n^2 + 2n + 1) y^n = 0$$

When  $x = 0$

$$(0-1) y^{(n+2)} + (2(0)n + 3(0) - n - 1) y^{(n+1)} + (n^2 + 2n + 1) y^n = 0$$

$$(-n-1) y^{(n+1)} + (n^2 + 2n + 1) y^n = 0$$

$$-(n+1) (y^{(n+1)})_0 + (n^2 + 2n + 1) (y^n)_0 = 0$$

$$(y^{(n+1)})_0 = \frac{(n^2 + 2n + 1) (y^n)_0}{n+1}$$

$$(y^{(n+1)})_0 = \frac{(n+1)(n+1)}{(n+1)} (y^n)_0$$

$$(y^{(n+1)})_0 = (n+1) (y^n)_0$$

When  $n = 0$

$$(y^{(0+1)})_0 = (0+1) (y^0)_0$$

$$(y^1)_0 = 1 (y^0)_0$$

When  $n = 1$

$$(y^{(1+1)})_0 = (1+1) (y^{(1)})_0$$

$$(y^{(2)})_0 = 2 (y^{(1)})_0$$

When  $n=2$

$$[y^{(3)}]_0 = (2+1) [y^{(2)}]_0$$

$$[y^{(3)}]_0 = 3[y^{(2)}]_0 = 3(2) [y^{(2)}]_0$$

$$[y^{(3)}]_0 = 6 [y^{(2)}]_0$$

When  $n=3$

$$[y^{(4)}]_0 = (3+1) [y^{(3)}]_0$$

$$[y^{(4)}]_0 = 4 [y^{(3)}]_0 = 4 [6] [y^{(2)}]_0$$

$$[y^{(4)}]_0 = 24 [y^{(2)}]_0$$

When  $n=4$

$$[y^{(5)}]_0 = (4+1) [y^{(4)}]_0$$

$$[y^{(5)}]_0 = 5 [y^{(4)}]_0 = 5(24) [y^{(2)}]_0$$

$$[y^{(5)}]_0 = 120 [y^{(2)}]_0$$

When  $n=5$

$$[y^{(6)}]_0 = (5+1) [y^{(5)}]_0$$

$$[y^{(6)}]_0 = 6 [y^{(5)}]_0 = 6 [120] [y^{(2)}]_0$$

$$[y^{(6)}]_0 = 720 [y^{(2)}]_0$$

When  $n=6$

$$[y^{(7)}]_0 = (6+1) [y^{(6)}]_0$$

$$= 7 [y^{(6)}]_0 = 7 (720) [y^{(2)}]_0$$

$$[y^{(7)}]_0 = 5040 [y^{(2)}]_0$$

$$y = [y^{(0)}]_0 + x [y^{(1)}]_0 + \frac{x^2}{2!} [y^{(2)}]_0 + \frac{x^3}{3!} [y^{(3)}]_0 + \frac{x^4}{4!} [y^{(4)}]_0$$

$$+ \frac{x^5}{5!} [y^{(5)}]_0 + \frac{x^6}{6!} [y^{(6)}]_0 + \frac{x^7}{7!} [y^{(7)}]_0$$

$$y = [y^{(0)}]_0 + x [y^{(1)}]_0 + \frac{x^2}{2!} \cdot 2 [y^{(2)}]_0 + \frac{x^3}{3!} \cdot 6 [y^{(3)}]_0 +$$

$$\frac{x^4}{4!} \cdot 24 (y^{(4)})_0 + \frac{x^5}{5!} \cdot 120 [y^{(5)}]_0 + \frac{x^6}{6!} = 720 [y^{(6)}]_0 +$$

$$\frac{x^7}{7!} 5040 (y^{(7)})_0$$

$$y (1+x)(y^{(1)})_0 + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) (y^{(1)})_0$$

$y(0) = 0.0005 \text{ m}, y'(0) = 0.0005$

$$y = (1+x)(0.0005 \text{ m}) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) (0.0005)$$

when  $x = 5 \text{ m}, 8 \text{ m}$  and  $10 \text{ m}$

$$y = (1+5)(0.0005 \text{ m}) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7) (0.0005)$$

$$y = 4 \times 3 \times 10^{-3} \text{ m} + 97650 \text{ m} (0.0005)$$

$$y = 3 \times 10^{-3} \text{ m} + 48.825 \text{ m}$$

$$y = 48.828 \text{ m}$$

when  $x = 8 \text{ m}$

$$y = (1+8)(0.0005) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7) (0.0005)$$

$$y = 4.5 \times 10^{-3} + 2396736 (0.0005)$$

$$y = 4.5 \times 10^{-3} + 1198$$

$$y = 1198 \text{ m}$$

when  $x = 10$

$$y = (1+10)(0.0005) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7) (0.0005)$$

$$y = 5.5 \times 10^{-3} + 11111100 (0.0005)$$

$$y = 5555.5555 \text{ m}$$

$$= 5556 \text{ m}$$

## Matlab

(1) Command Window

(2) Clear

(3) CLC

(4) Close all

(5) Syms x

6  $x = ((1+x) * (0.0005)) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) *$   
7  $(0.0005)$

8  $t = 0:0.01:10$

9  $xt = \text{subs}(x, t)$

10  $xt_n = \text{double}(xt)$

11  $\text{plot}(t, xt_n)$

12  $x \text{ label}('t')$

13  $y \text{ label}('x')$

14  $\text{grid on}$

15  $\text{grid minor}$

16  $\text{axis tight}$