7) $x(x-1)y'' + (3x-1)y' + y = 0$

Expanding the bracket

$(x^2 - x)y'' + (3x-1)y' + y = 0$

$W_1 = (x^2 - x)y''$

$W_2 = (3x-1)y'$

$W_3 = y$

~~Using Leibnitz theorem; $u^n v + n u^{n-1} v^{-1} + n(n-1)(n-2) u^{n-3} v^3 +$~~

For W_1		For W_2		For W_3	
$u = y^2$	$v = x^2 - x$	$u = y'$	$v = 3x-1$	$u = y$	$v = 1$
$u^n = y^{n+2}$	$v' = 2x-1$	$u^n = y^{n+1}$	$v' = 3$	$u^n = y^n$	$v' = 0$
$u^{n-1} = y^{n+1}$	$v^2 = 2$	$u^{n-1} = y^n$	$v^2 = 0$		
~~u^n~~ $u^{n-2} = y^n$	$v^3 = 0$				

Using Leibnitz theorem; $u^n v + n u^{n-1} v^{(1)} + \frac{n(n-1)}{2!} u^{n-2} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^{(3)} + \dots$

$\therefore W_1 = y^{(n+2)} \cdot (x^2 - x) + n y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2} y^n \cdot 2$

$= (x^2 - x) y^{(n+2)} + n(2x-1) y^{(n+1)} + n(n-1) y^n$

$W_2 = y^{(n+1)} \cdot (3x-1) + 3n y^n$

$W_3 = y^n$

$W_1 + W_2 + W_3 = 0$

$(x^2 - x) y^{(n+2)} + n(2x-1) y^{(n+1)} + n(n-1) y^n + y^{(n+1)} \cdot (3x-1) + 3n y^n = 0$

Assuming $x = 0$

$= -n y^{(n+1)} + (n^2 - n) y^n - y^{(n+1)} + 3n y^n + y^n = 0$

Collecting Like terms

$= y^{(n+1)}(-n-1) + y^n(n^2 - n + 3n + 1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2 + 2n + 1) = 0$

$(n+1) y^{(n+1)} = (n^2 + 2n + 1) y^n$

$\therefore (n+1) y^{(n+1)} = (n+1)(n+1) y^n$

Divide both sides by $(n+1)$

$y^{(n+1)} = (n+1) y^n$ ... Recurrence relation

$(y^{n+1})_0 = y^n(n+1)$

$(y^0)_0 = 0.0005$

$(y')_0 = 0.0005$

when $n = 0$

$[y^{(0+1)}]_0 = (0+1)(y^0)_0$

~~$[y^{(1)}]_0$~~ $[y^{(1)}]_0 = 1[y^0]_0$

$n=1;$ $[y^{(1+1)}]_0 = (1+1)(y')_0$

$[y^{(2)}]_0 = 2(y')_0$

$n=2;$ $(y^3)_0 = (2+1)y^{(2)}$

$= 3[y^2]_0 = 3[2(y^{(1)})']$

$n=3;$ $(y^4)_0 = (3+1)y^3$

$= 4(y^3)_0 = 4[6(y')_0] = 24(y')_0$

$n=4;$ $(y^5)_0 = (4+1)y^4$

$= 5(y^4)_0 = 120(y')_0$

$n=5;$ $(y^6)_0 = (5+1)y^5$

$= 6(y^5)_0 = 720(y')_0$

$n=6;$ $(y^7)_0 = (6+1)y^6$

$= 7(y^6) = 5040(y')_0$

Using Mauclarin Series

$$y = (y^0)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$$

$$y = (y^0)_0 + x(y^0) + \frac{x^2}{2!}(2y')_0 + \frac{x^3}{3!}(6y')_0 + \frac{x^4}{4!}(24y')_0 + \frac{x^5}{5!}(120y') + \frac{x^6}{6!}(720y')_0 + \frac{x^7}{7!}(5040y')$$

$= y^0(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)y'$

$= 0.0005(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)\,0.0005$

ii) Estimate the ~~approxciate~~ approximate deformation when $x = 5, 8$ and $10m$

When $x = 5m$

Scanned by CamScanner

$y = y^0(1+5) + (5^2+5^3+5^4+5^5+5^6+5^7)0.0005$

$= 0.0005(6) + (25+125+625+3125+15625+78125)\ 0.0005$

$y = 48.828m$

when $x = 8m$

$y = y^0(1+8) + (8^2+8^3+8^4+8^5+8^6+8^7)y'$

$y = 0.0005(1+8) + (64+512+4096+32768+262144+2097152)0.0005$

$= 1198.3725m$

when $x = 10m$

$y = y^0(1+10) + (10^2+10^3+10^4+10^5+10^6+10^7)y'$

$= 0.0005(11) + (100+1000+10\,000+100\,000+1000\,000+10000\,000)0.0005$

$= 5555.56m$

MATLAB mfile

Command Window

```
clear
clc
close all
x =0:0.01:10
y = (0.0005*(1+x)) + ((x^2 +x^3 + x^4 + x^5 +x^6+x^7)*0.005)
yn = subs(y)
plot (x,yn)
x label ('m')
y label ('Deflection')
axis tight
grid on
grid minor
```

SKETCH (GRAPH OF STRUCTURAL ELEMENT AGAINST x