NAME : Okike Chukwuyameah
~~Department~~ MATRIC NO: 17/Eng02/065
Department: Computer Engineering

ENG 381 ASSIGNMENT 3

The model for deformation (y) of a structural element is represented by the expression given in equation (1)

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz - Maclaurin method

a) Obtain the power series solution of the model up to and including the terms in x^2

b) estimate the appropriate deformation when $x = 5$, 8 and 10m, and

c) with the aid of a MATLAB m-file program, plot the response of the structural element for $0 \le x \le 10m$

Solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Expanding the bracket

$$(x^2 - x)y'' + (3x-1)y' + y = 0$$

$$w_1 = (x^2 - x)y''$$

$$w_2 = (3x-1)y'$$

$$w_3 = y$$

Using Leibnitz theorem

for w_1

$$U^n V + nU^{n-1}V^1 + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3 + \ldots$$

for w_1	for w_2	for w_3
$U = y^2$ $V = x^2 - x$	$U = y'$ $V = 3x-1$	$U = y$ $V = 1$
$U^n = y^{n+2}$ $V' = 2x-1$	$U^n = y^{n+1}$ $V' = 3$	$U^n = y^n$ $V' = 0$
$U^{n-1} = y^{n+1}$ $V^2 = 2$	$U^{n-1} = y^n$ $V^2 = 0$	
$U^{n-2} = y^n$ $V^3 = 0$		

$W_1 = y^{(n+2)} \cdot (x^2 - x) + n \cdot y^{(n+1)} (2x - 1) + \frac{n(n-1)}{2!} \cdot y^n \cdot \cancel{2}$

$W_2 = y^{(n+1)} \cdot (3x-1) + 3n y^n$

$W_3 = y^n$

Summing all together.

$$y^{(n+2)} \cdot (x^2 - x) + (2nx - n) y^{(n+1)} + y^n (n^2 - n) + y^{(n+1)} (3x-1) + 3n y^n + y^n = 0$$

assuming $x = 0$

$$y^{(n+2)} \cdot \cancel{(0^2 - 0)} + (2n(0) - n) y^{(n+1)} + y^n (n^2 - n) + y^{(n+1)} (3(0) - 1) + 3n y^n + y^n = 0$$

$$= -n y^{(n+1)} + y^n (n^2 - n) - y^{(n+1)} + 3n y^n + y^n = 0$$

Collecting like terms

$$= y^{(n+1)} (-n-1) + y^n (n^2 - n + 3n + 1) = 0$$

$$= y^{(n+1)} (-n-1) + y^n (n^2 + 2n + 1) = 0$$

$$= -y^{n+1} (n+1) + y^n (n^2 + 2n + 1) = 0$$

$$= y^{(n+1)} (n+1) = y^n \underbrace{(n^2 + 2n + 1)}_{(n+1)(n+1)}$$

$$\therefore y^{(n+1)} (n+1) = y^n (n+1)(n+1)$$

Divide both side by $(n+1)$

$$\frac{y^{(n+1)} \cancel{(n+1)}}{\cancel{(n+1)}} = \frac{y^n (n+1) \cancel{(n+1)}}{\cancel{(n+1)}}$$

$$\boxed{y^{(n+1)} = y^n (n+1)}$$ — Recurrence relation.

$$\left(y^{n+1}\right)_0 = y^n (n+1).$$

$(y^0)_0 = 0.0005$

$(y^1)_0 = 0.0005$

when n=0

$$[\gamma^{(0+1)}]_0 = (0+1)(\gamma^0)_0$$

$$[\gamma^{(1)}]_0 = 1[\gamma^0]_0$$

When n=1

$$[\gamma^{(1+1)}]_0 = (1+1)(\gamma^1)_0$$

$$[\gamma^{(2)}]_0 = 2(\gamma^1)_0$$

when n=2

$$(\gamma^3)_0 = (2+1)\gamma^2$$

$$(\gamma^3)_0 = 3[\gamma^2]_0 = 3(2)[\gamma^{(1)}]_0$$

$$(\gamma^{(3)})_0 = 6(\gamma^1)_0$$

When n=3

$$(\gamma^4)_0 = (3+1)\gamma^3$$

$$(\gamma^4)_0 = 4[\gamma^3]_0 = 4[6(\gamma^1)_0] = 24(\gamma^1)_0$$

when n=4

$$(\gamma^5)_0 = (4+1)\gamma^4$$

$$(\gamma^5)_0 = 5(\gamma^4) = 5[24(\gamma^1)_0] = 120(\gamma^1)_0$$

when n=5

$$\gamma^6 = (5+1)\gamma^5$$

$$(\gamma^6)_0 = 6(\gamma^5) = 6[120(\gamma^1)_0) = 720(\gamma^1)_0$$

When n=6

$$\gamma^7 = (6+1)\gamma^6$$

$$(\gamma^7)_0 = 7(\gamma^6) = 7(720(\gamma^1)_0) = 5040(\gamma^1)_0$$

$\therefore \gamma_{1/2}$

Using Maclaurin Series:

$$y = (y^0)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5) + \frac{x^6}{6!}(y^6) + \frac{x^7}{7!}(y^7)$$

$$y = (y^0)_0 + x(y^0)_0 + \frac{x^2}{\cancel{2}!}(\cancel{2}y')_0 + \frac{x^3}{\cancel{3!}}(\cancel{6}y')_0 + \frac{x^4}{\cancel{4!}}(\cancel{24}y') + \frac{x^5}{\cancel{5!}}(\cancel{120}y')$$

$$+ \frac{x^6}{\cancel{6!}}(\cancel{720}y') + \frac{x^7}{\cancel{7!}}(\cancel{5040}y')$$

$$y = y^0(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)\,y'$$

$$\therefore y = 0.0005(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)\,0.0005$$

ii. Estimate the approximate deformation when $x = 5, 8$ and 10 m

when $x = 5$ m

$$y = y_0(1+5) + (5^2+5^3+5^4+5^5+5^6+5^7)\,0.0005$$

$$y = 0.0005(1+5) + (25+125+625+3125+15625+78125)\,0.0005$$

$$y = 48.828 \text{ m}$$

when $x = 8$ m

$$y = y_0(1+8) + (8^2+8^3+8^4+8^5+8^6+8^7)\,0.0005$$

$$y = 0.0005(1+8) + (64+512+4096+32768+262144+2097152)\,0.0005$$

$$y = 1198.3725 \text{ m}$$

when $x = 10$ m

$$y = y_0(1+10) + (10^2+10^3+10^4+10^5+10^6+10^7)\,y'$$

$$y = 0.0005(1+10) + (100+1000+10000+100000+1000000+10000000)\,0.0005$$

$$y = 5555.56 \text{ m}$$

MATLAB mfile

Command window

```
Ckar
clc
close all
x = 0:0.01:10
a) y = (0.0005*(1+x)) + ((x.^2+x.^3+x.^4+x.^5+x.^6+x.^7)*0.0005)
b) yn = subs(y)
c) plot (x, yn)
xlabel('m')
ylabel('Deflection')
axis tight
grid on
grid minor
```

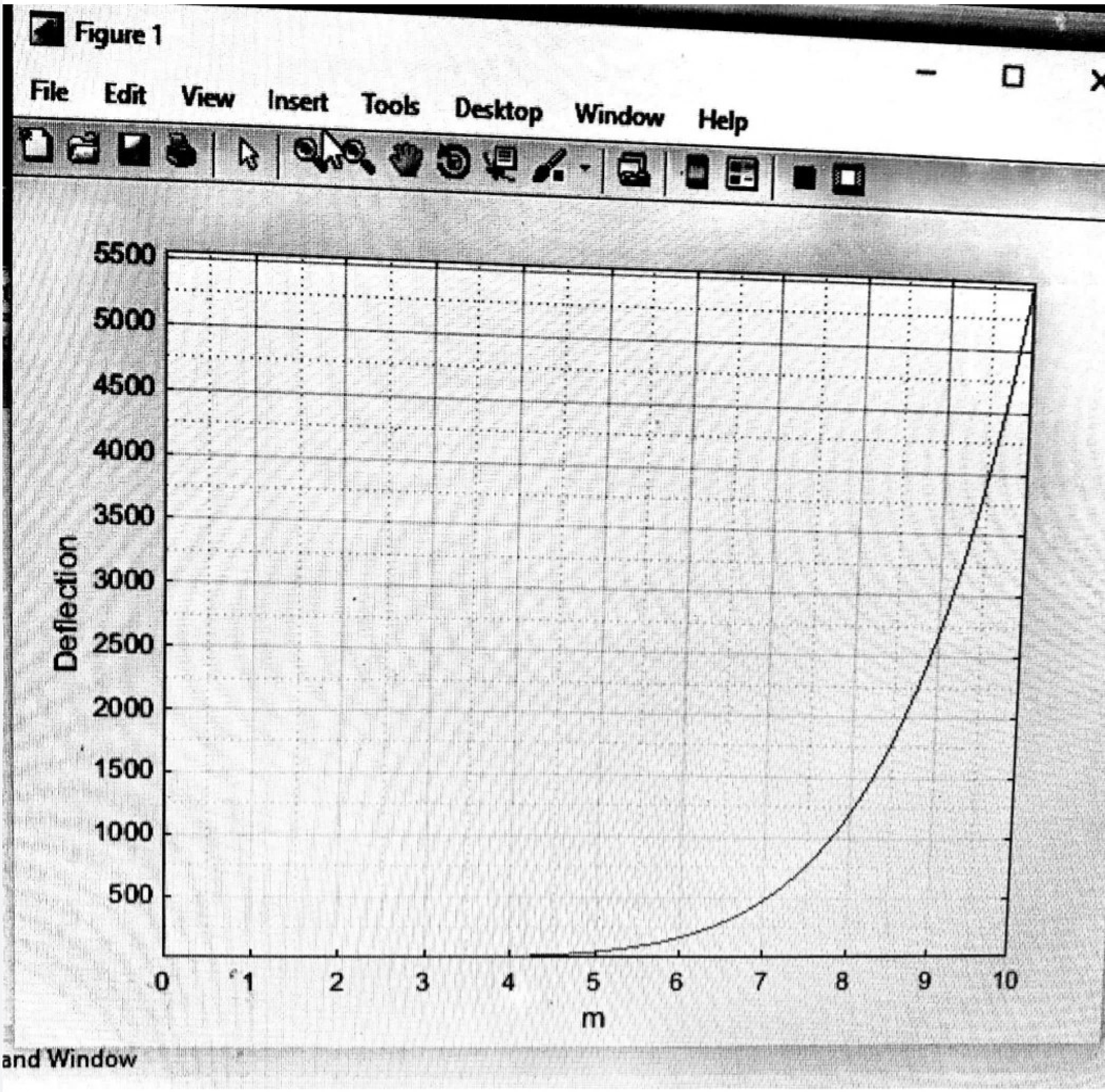
Figure 1
File Edit View Insert Tools Desktop Window Help
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
Deflection
0 1 2 3 4 5 6 7 8 9 10
m
and Window