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Electrical Electronics Engineering
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Assignment

The model for the deformation (y) of a structural element is represented by the expression given in equation (1) $x(x-1)y'' + (3x-1)y' + y = 0$. Given that $y(0) = 0.0005$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin method

a. Obtain the power series solution of the model up to and including the term in x

b. Estimate the appropriate deformation when $x = 5, 8$ and 10

c. With the aid of a matlab mfile program, plot the response of the structural element for $0 \le x \le 10m$

Solution

Using Leibnitz method

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$(x^2 - x)y'' + (3x-1)y' + y = 0$$

$$(x^2-x)y^{n+2} + n(2x-1)y^{n+1} + \frac{n(n-1)}{2!}2y^n + (3x-1)y^{n+1} + n(3)y^n + y^n = 0$$

$(x^2-x)y^{n+2}+n(2x-1)y^{n+1}+n(n-1)y^n+(3x-1)y^{n+1}+3ny^n+y^n=0$

$(x^2-x)y^{n+2}+(2nx-n)y^{n+1}+(n^2-n)y^n+(3x-1)y^{n+1}+3ny^n+y^n=0$

$(x^2-x)y^{n+2}+(2nx-n+3x-1)y^{n+1}+(n^2-n+3n+1)y^n=0$

$(x^2-x)y^{n+2}+(2nx-n+3x-1)y^{n+1}+(n^2+2n+1)y^n=0$

when $x=0$

$(2n(0)-n+3(0)-1)y^{n+1}+(n^2+2n+1)y^n=0$

$(-n-1)y^{n+1}+(n^2+2n+1)y^n=0$

$+(n+1)y^{n+1}=(n^2+2n+1)y^n$

$$y^{n+1}=\frac{(n^2+2n+1)y^n}{(n+1)}$$

$$(y^{n+1})_0=\frac{(n+1)(n+1)(y^n)_0}{(n+1)}$$

$(y^{n+1})_0=(n+1)(y^n)_0$

at $n=0$; $y^1=1y^0$

$n=1$, $y^2=2(y^1)_0$

$n=2$; $y^3=3y^2=3(2)y^1=6(y^1)_0$

$n=3$; $y^4=4y^3=4(3)(2)y^1=24(y^1)_0$

$n=4$; $y^5=5y^4=5(4)(3)(2)y^1=5!(y^1)_0$

$n=5$; $y^6=6y^5=6(5)(4)(3)(2)y^1=6!(y^1)_0$

Maclaurin Series

$y = y_0 + x(y'_0) + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$

$= y^0 + x(y'_0) + \frac{x^2}{2!}(2y'_0) + \frac{x^3}{3!}(3!y'_0) + \frac{x^4}{4!}(4!y'_0) + \frac{x^5}{5!}(5!y'_0) + \frac{x^6}{6!}(6!y'_0) + \frac{x^7}{7!}(7!y'_0)$

$= y^0 + x(y')_0 + x^2(y')_0 + x^3(y')_0 + x^4(y')_0 + x^5(y')_0 + x^6(y')_0 + x^7(y')_0$

$= y^0 + y!(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

Recall

$y' = y^0$

$y^0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$y' = 0.0005$ and $y^0 = 0.0005$ when $x = 5$

$y^5 = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$= 0.0005(97656) = 48.828 \simeq 49$

when $x = 8$, $y_0 = 0.0005$

$y_8 = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$= 0.0005(2396745)$

$= 1198.3725 \simeq 1198$

$x = 10$

$y_{10} = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

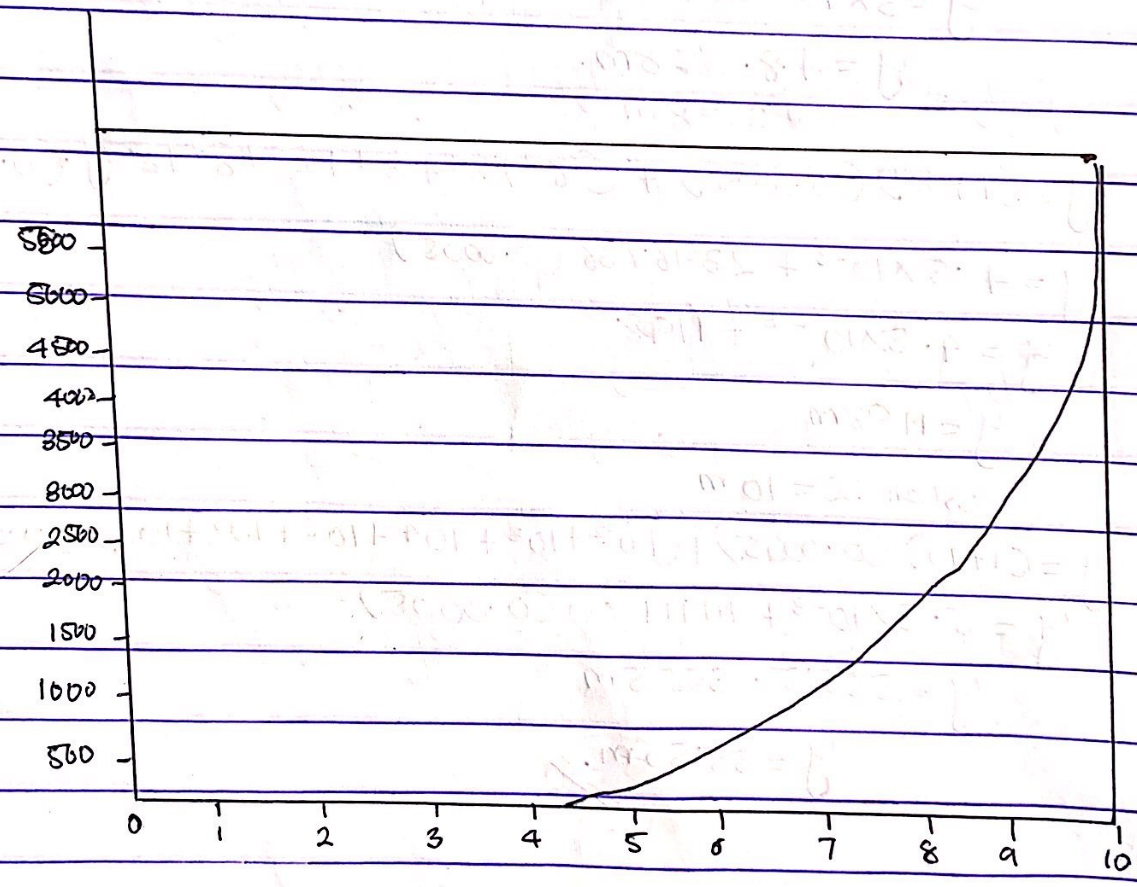
$= 0.0005(11111111)$

$= 5555.5555 \simeq 5556$

c Command window

```
clc
clear all
close all
syms x y
x = 0:0.1:10
y = (0.0005)*(1+x+x^2+x^3+x^4+x^5+x^6+x^7)
yn = subs(y)
ynn = double(yn)
plot(x,ynn)
xlabel('x')
ylabel('T')
grid on
grid minor
axis tight
```

Scanned with CamScanner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