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18/ENG06/012
MECHANICAL ENGINEERING

QUESTION

① Find the area bounded by the curve $y = 3e^{2x}$ and $y = 3e^{-x}$ and the coordinates at $x = 1$ and $x = 2$.

② The parametric equation of a curve are $y = 2\sin \pi/10t$ and $x = 2 + 2t - 2\cos \pi/10t$. Find the area under the curve between $t = 0$ and $t = 10$

SOLUTION

① $y = 3e^{2x}$

$x = 3e^{-x}$

$$A = \int_{x_1}^{x_2} f(x) - g(x)\, dx$$

$$= \int_1^2 3e^{2x} - 3e^{-x}\, dx$$

$$= 3\int_1^2 e^{2x} - e^{-x}\, dx$$

$$= 3\int_1^2 \frac{e^{2x}}{2} - e^{-x}$$

$$= 3\left[\frac{e^{2(2)}}{2} + e^{(-2)}\right] - 3\left[\frac{e^{2(1)}}{2} + e^{(-1)}\right]$$

$$= [3 \times 27.4344103] - [3 \times 4.0624]$$

$$= 82.30 - 12.18$$

$$= 70.12 \text{ square unit.}$$

② $y = 2 \sin \frac{\pi}{10} t$

$x = 2 + 2t - \frac{2\cos \pi}{10} t$

$$A = \int_{t_1}^{t_2} y(7) - \frac{dx}{dt} \cdot dt$$

$$A = \int_{10}^{10} (2 \sin \frac{\pi}{10} t \times 2) + \left[\cancel{2+2t} \; \frac{2\pi}{5} \; \frac{\sin \pi}{10} \; dt \right]$$

$$A = \int_{10}^{10} 4 \left[\sin \frac{\pi}{10} t \right] \left[1 + \frac{2\pi}{5} \sin \frac{\pi}{10 t} \; dt \right]$$

$$A = 4 \int_{10}^{10} \frac{\sin \pi}{10} + \frac{2\pi}{5} \left[1 - \frac{\cos \pi}{\frac{5t}{2}} \right] dt$$

INTEGRATION BY PART

$$A = 4 \int_{10}^{10} \sin \frac{\pi}{10} + \frac{2\pi}{5} \int_{10}^{10} \left[1 - \frac{\cos \pi}{\frac{5t}{2}} \right] dt$$

$$A = 4 \left[-\frac{10}{\pi} \cos \frac{\pi}{10} (10) + \frac{4}{5} \left[10 - \frac{5}{\pi} \sin \frac{\pi}{5} (10) \right] - 4 \left(\frac{-10}{5} \frac{\cos \pi(0)}{10} \right. \right.$$

$$\left. \left. + \frac{\pi}{5} \left[\frac{0.5}{\pi} \sin \frac{\pi}{5} [0] \right] \right. \right.$$

$A = (12.73 + 2\pi - (-12.73 + 0)$

$= 31.744$ square unit