Elect/Elect

1) $x(x-1)y'' + (3x-1)y' + y = 0$

$x(x-1)y^2 + (3x-1)y' + y = 0$

$W_1 = x(x-1)y^2 \quad - ①$

let $u = y^2$ $\quad v = x(x-1),\ x^2 - x$

$u^n = y^{n+2}$ $\quad v' = 2x-1$

$u^{n-1} = y^{n-1}$ $\quad v^2 = 2$

$u^{n-2} = y^n$ $\quad v^3 = 0$

$W_2 = (3x-1)y' \quad - ⓘ$

$u = y'$ $\quad v = 3x-1$

$u^n = y^{n-1}$ $\quad v' = 3x$

$u^{n-1} = y^n$ $\quad v^2 = 0$

$W_3 = y \quad - (14)$

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0.$

Applying Leibnitz in eqn ①, ⓘ & (14)

① $y^n = u^n v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \frac{n(n-1)(n-2)(n-3)}{4!}$

$u^{n-4} v^4 + \ldots$

$y^n = y^{n+2}(x^2 - x) + n x y^{n+1}(2x-1) + \frac{n(n-1)}{2} \times y^n\, 2 + 0$

$y^n = x(x-1)y^{(n+2)} + (2x-1)n y^{n+1} + n(n-1)y^n \quad - ①$

ⓘ $y^n = y^{n-1}(3x-1) + n y^n (3) + 0$

(14) $y^n = y^n \times 1 + 0 = y^n$

~~Assuming, $x=0$~~

~~$-n(y^{n+1})_0 + n(n-1)y^n + (3x-1)y^{n+1} + 3n y^{(n)} + y^n = 0$~~

$x(x-1)y^{(n+2)} + (2x-1)n y^{(n+1)} + n(n-1)y^n + (3x-1)y^{n+1} + 3n y^n + y^n = 0$

Assuming $x = 0$

$-n(y^{n+1})_0 + n(n-1)(y^n)_0 - (y^{n+1})_0 + 3n(y^n)_0 + (y^n)_0 = 0$

$-(y^{n+1})_0 (n+1) + (y^n)_0 [n(n-1) + 3n + 1] = 0$

$-[y^{n+1}]_0 (n+1) + (y^n)_0 (n^2 + 2n + 1) = 0$

$+(n+1)(y^{n+1})_0 = +(n^2 + 2n + 1)(y^n)_0$

$(n+1)(y^{n+1})_0 = (n^2 + 2n + 1)(y^n)_0$

when $n = 0$

$(y^n)_0 = (y^0)_0$

But $(y^0)_0 = 0.0008m$

$(y')_0 = 0.0008$

when $n = 1$

$2(y^2)_0 = (1+2+1)(y')_0$

$2(y'')_0 = 4(y')_0$

$(y'')_0 = 2(y')_0$

$= 2 \times 0.0005 = 1 \times 10^{-3}$

when $n = 2$

$3(y''')_0 = (4+4+1)(y^2)_0$

$3(y^3)_0 = 9(y^2)_0$

$(y^3)_0 = 3(y^2)_0$

$(y^3)_0 = 3 \times (1 \times 10^{-3}) = 3 \times 10^{-3}$

when $n = 3$;

$4(y^4)_0 = (9+6+1)(y^3)_0$

$(y^4)_0 = 4(y^3)_0$

$= 4 \times (3 \times 10^{-3}) = 0.012 / 12 \times 10^{-3}$

when $n = 4$;

$5(y^5)_0 = (16+8+1)(y^4)_0$

$(y^5)_0 = 5(y^4)_0$

$= 5 \times 0.012 = 0.06$,

when $n = 5$,

$(y^6)_0 = 6(y^5)_0$

$= 6 \times 0.06 = 0.36$

when $n = 6$;

$(y^7)_0 = 7(y^6)_0$

$= 7 \times 0.36 = 2.52$

Using Leibnitz Eqn

$y = (y^0)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6)_0 + \frac{x^7}{7!}(y^7)_0$

$y = 0.0005 + 0.0005x + \frac{x^2}{2!}(1 \times 10^{-3}) + \frac{x^3}{3!}(3 \times 10^{-3}) + \frac{x^4}{24} 0.012 \times \frac{x^5}{120} 0.06 + \frac{x^6}{720} \times 0.36 + \frac{x^7}{5040} \times 2.52$

b) When $x = 5$

$y = 5\times10^{-4}(1+5+25+125+625+3125+15625+78125)$

$y = 5\times10^{-4} \times 97656$

$y = 48.828$

When $x = 8$

$y = 5\times10^{-4}(1+8+64+512+4096+32768+262144+2097152)$

$y = 1198.3725$

When $x = 10$

$y = 5\times10^{-4}(1+10+100+1000+10{,}000+10^5+10^6+10^7) = 5555.5555$

c) ~~Notes~~

Command Window

```
Clear
Clc
close all
Sysms x
Y = (5*10^-4) * (1+x+x.^2+x.^3+2^4+x.^5+x.^6+x.^9)
x = (0:10)
Yn = Subs(Y,x)
  Plot(n,Yn)
x label('Structural Element (m)')
y label('Deformation')
Grid on
Grid minor
axis tight
```

$\ldots + \frac{x^6}{6!}(y^6)_0 +$

$\frac{x^5}{120}$ 0.06 +